AP Calculus BC
Unit 11 – REVIEW

Name: Answer Key *

1. For time $t > 0$, the position of a particle moving in the xy-plane is given by the parametric equations $x(t) = 4t + t^2$ and $y(t) = \dfrac{1}{3t+1}$. What is the acceleration vector of the particle at $t = 1$?

$\langle 2, 0.281 \rangle$

2. Find the area of the common interior of the polar curves $r = 4\cos\theta$ and $r = 2$.

4.912

3. Write an equation for the line tangent to the polar curve $r = 2\theta$ at $\theta = \dfrac{\pi}{2}$.

$y - \pi = \dfrac{-2}{\pi}x$

4. Find the equation of the tangent to the curve defined by $x = \sqrt{t}$ and $y = \sqrt{t-1}$ when $t = 5$.

$y - 2 = \dfrac{\sqrt{5}}{2}(x - \sqrt{5})$

5. Find $\dfrac{d^2y}{dx^2}$ for the curve given by $x = \dfrac{1}{2}t^2$ and $y = t^2 + t$.

$\dfrac{-1}{t^3}$

6. Find all points of vertical tangency to the curve given by $x = \cos\theta$ and $y = 4\sin\theta$.

$(1, 0)$ & $(-1, 0)$

7. Find the total distance a particle travels along a path by $x = t^2 + 1$ and $y = 4t + 3$ on the interval $-1 \le t \le 0$.

4.161

8. The position of a particle in the xy-plane is given by (x(t), y(t)), with $\frac{dy}{dt} = t^2 + \cos(3t^2)$. At t = 0, the particle is at the point (3, 1). Find the y-coordinate of the particle at t = 3.

10.415

9. A particle follows a path defined parametrically by $x(t) = 2\sqrt{t-3}$ and $y(t) = 3t^2$. What is the speed of the particle at t = 9?

54.002

10. Find the area of the region enclosed by the graph of $x = \sin\theta, y = \sin^2\theta$, the x-axis, and the vertical line x = 1.

0.333

11. Find $\frac{dy}{dx}$ for $r = 3(1-\cos\theta)$ at $\theta = \pi$.

undefined

12. Find the tangents at the pole for the polar curve $r = 2\cos 3\theta$.

$\pi/6$, $\pi/2$, $5\pi/6$

13. Find the perimeter of one petal of the rose curve $r = 4\sin(3\theta)$.

8.910

14. Find the points of intersection of the graphs of $r = 2-3\cos\theta$ and $r = \cos\theta$.

$(0,0)$ $(\frac{1}{2}, \pi/3)$ $(\frac{1}{2}, 5\pi/3)$

15. Find the area of the common interior region of $r = 4\sin\theta$ and $r = 2$.

4.912

① $x(t) = 4t + t^2$ $\quad y(t) = \frac{1}{3t+1} = (3t+1)^{-1}$

acceleration vector ? at $t=1$.

$x'(t) = 4+2t$ $\quad y'(t) = -1(3t+1)^{-2}(3)$

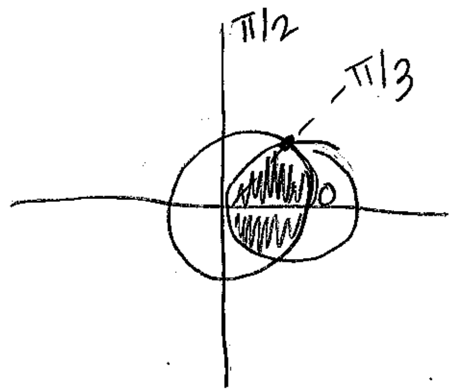
$y'(t) = -3(3t+1)^{-2}$

$\langle x''(1), y''(1) \rangle$ use math 8

$\boxed{\langle 2, 0.281 \rangle}$

② Area of common interior

$r = 4\cos\theta \quad r = 2$



$4\cos\theta = 2$

$\cos\theta = 1/2$

$\theta = \pi/3$

$A = \frac{1}{2}\int_0^{\pi/3} (2)^2 \, d\theta = 2.094$

$A = \frac{1}{2}\int_{\pi/3}^{\pi/2} (4\cos\theta)^2 \, d\theta = 0.362$

$2.094 + 0.362 = 2.456$

$2.456(2) = \boxed{4.912}$

③ equation of tangent line

$r = 2\theta$ at $\theta = \pi/2$

$x = r\cos\theta \qquad y = r\sin\theta$

$x = 2\theta\cos\theta \qquad y = 2\theta\sin\theta$

$x = \cancel{2}(\tfrac{\pi}{\cancel{2}})\cos(\tfrac{\pi}{2}) \qquad y = \cancel{2}(\tfrac{\pi}{\cancel{2}})(\sin\tfrac{\pi}{2})$

$x = 0 \qquad y = \pi$

$(0, \pi)$ point

slope:

$$\frac{dy}{dx} = \frac{(2)(\sin\theta) + (2\theta)(\cos\theta)}{(2)(\cos\theta) + (2\theta)(-\sin\theta)}$$

$$\frac{2\sin\pi/2 + \cancel{2(\tfrac{\pi}{2})\cos(\tfrac{\pi}{2})}}{2\cancel{\cos(\pi/2)} + \cancel{2}(\tfrac{\pi}{\cancel{2}})(-\sin(\tfrac{\pi}{2}))} = \frac{2}{-\pi}$$

$y - \pi = -\frac{2}{\pi}(x-0)$

$$\boxed{y - \pi = -\frac{2}{\pi}x}$$

④ eqn of tangent line $x = \sqrt{t} \quad y = \sqrt{t-1}$ when $t = 5$

$x = t^{1/2} \quad y = (t-1)^{1/2}$

slope: $\frac{dy}{dx} = \frac{\cancel{\frac{1}{2}}(t-1)^{-1/2}}{\cancel{\frac{1}{2}}t^{-1/2}} = \frac{\sqrt{t}}{\sqrt{t-1}} \rightarrow$ at $t=5$ $\frac{\sqrt{5}}{\sqrt{5-1}} = \frac{\sqrt{5}}{2}$

point: $x = \sqrt{5} \qquad y = \sqrt{5-1} = \sqrt{4} = 2$

$(\sqrt{5}, 2)$

$$\boxed{y - 2 = \frac{\sqrt{5}}{2}(x - \sqrt{5})}$$

5) $\frac{d^2y}{dx^2}$? $x = \frac{1}{2}t^2$ $y = t^2 + t$

$$\frac{dy}{dx} = \frac{2t+1}{t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(t)(2)-(2t+1)(1)}{t^2}}{t} = \frac{\cancel{2t}-\cancel{2t}-1}{t^3} = \boxed{-\frac{1}{t^3}}$$

6) vert. tangency $x = \cos\theta$ $y = 4\sin\theta$

↓

$$\frac{dx}{dt} = 0$$

$$-\sin\theta = 0$$

$$\sin\theta = 0$$

$$\theta = 0, \pi$$

$\underline{\theta = 0}$: $x = \cos 0 = 1$, $y = 4\sin 0 = 0$ $\boxed{(1,0)}$

$\underline{\theta = \pi}$: $x = \cos\pi = -1$, $y = 4\sin\pi = 0$ $\boxed{(-1,0)}$

7) Total distance $x = t^2+1$ $y = 4t+3$ $-1 \leq t \leq 0$

"Arc Length"

$$= \int_{-1}^{0} \sqrt{(2t)^2 + (4)^2}\, dt = \boxed{4.161}$$

(8) $\frac{dy}{dt} = t^2 + \cos(3t^2)$

$x(0) = 3 \quad y(0) = 1 \quad y(3) = ?$

$y(3) - y(0) = \int_0^3 t^2 + \cos(3t^2)\,dt$

$y(3) - 1 = 9.415$

$\boxed{y(3) = 10.415}$

(9) $x(t) = 2\sqrt{t-3} \quad y(t) = 3t^2$ speed at $t = 9$?

$x(t) = 2(t-3)^{1/2}$

$x'(t) = (t-3)^{-1/2} \quad y'(t) = 6t$

speed $= \sqrt{((9-3)^{-1/2})^2 + (6 \cdot 9)^2} = \boxed{54.002}$

(10) Area $x = \sin\theta \quad y = \sin^2\theta$ x-axis, vertical line $x = 1$

$dx = \cos\theta\, d\theta$

$A = \int_a^b y\, dx$

$x = 0$ & $x = 1$

$x = \sin\theta \quad x = \sin\theta$

$0 = \sin\theta \quad 1 = \sin\theta$

$\theta = 0 \quad \theta = \pi/2$

$= \int_0^{\pi/2} \sin^2\theta (\cos\theta)\, d\theta$

$= \boxed{0.333}$

(11) $\frac{dy}{dx}$ $\quad r=3(1-\cos\theta)$ at $\theta=\pi$.

$r=3-3\cos\theta$

$x=r\cos\theta$ $\qquad y=r\sin\theta$

$x=(3-3\cos\theta)(\cos\theta)$ $\qquad y=(3-3\cos\theta)(\sin\theta)$

$$\frac{dy}{dx}=\frac{(3\sin\theta)(\sin\theta)+(3-3\cos\theta)(\cos\theta)}{(3\sin\theta)(\cos\theta)+(3-3\cos\theta)(-\sin\theta)}$$

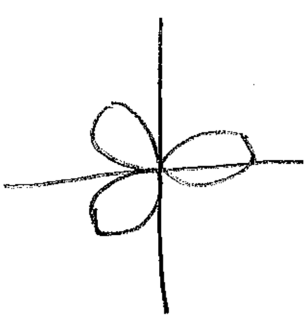
$\cos\pi=-1$

$\sin\pi=0$

$$=\frac{\cancel{(3\sin\pi)(\sin\pi)}+(3-3\cos\pi)(\cos\pi)}{\cancel{(3\sin\pi)(\cos\pi)}+\cancel{(3-3\cos\pi)(-\sin\pi)}}$$

$$=\frac{(3-3(-1))(-1)}{0}=\boxed{\text{Undefined}}$$

(12) Tangents at pole $\quad r=2\cos3\theta$



$0\le\theta<\pi$

$0\le3\theta<3\pi$

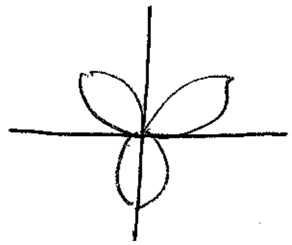
$2\cos3\theta=0$

$\cos3\theta=0$

$3\theta=\frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2}$

$\boxed{\theta=\frac{\pi}{6}}$ $\quad\theta=\frac{3\pi}{6}$ $\quad\boxed{\frac{\pi}{2}}$ $\quad\boxed{\frac{5\pi}{6}}$

(13) Perimeter one petal $r = 4\sin(3\theta)$



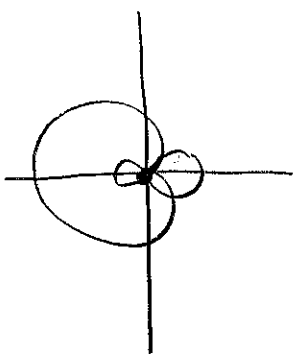
$$\int_0^{\pi/3} \sqrt{(4\sin 3\theta)^2 + (12\cos(3\theta))^2}$$

$$= \boxed{8.910}$$

$0 \le \theta < \pi$

one petal $\rightarrow$ $0 \le \theta < \pi/3$

(14) Pts of Intersection $r = 2 - 3\cos\theta$ $r = \cos\theta$



intersects at pole $\rightarrow$ $\boxed{(0,0)}$

$2 - 3\cos\theta = \cos\theta$

$2 = 4\cos\theta$

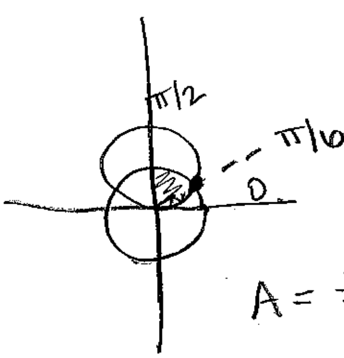
$\frac{1}{2} = \cos\theta$

$\theta = \pi/3, 5\pi/3$

$r = \cos(\pi/3) = 1/2$ $\boxed{(\frac{1}{2}, \pi/3)}$

$r = \cos(5\pi/3) = 1/2$ $\boxed{(\frac{1}{2}, \frac{5\pi}{3})}$

(15) Area $r = 4\sin\theta$ $r = 2$



$4\sin\theta = 2$

$\sin\theta = 1/2$

$\theta = \pi/6$

$$\left.\begin{aligned} A &= \tfrac{1}{2}\int_0^{\pi/6} (4\sin\theta)^2\,d\theta = 0.362 \\ A &= \tfrac{1}{2}\int_{\pi/6}^{\pi/2} (2)^2\,d\theta = 2.094 \end{aligned}\right\}$$

$.362 + 2.094 = 2.456$

$2.456(2) = \boxed{4.912}$