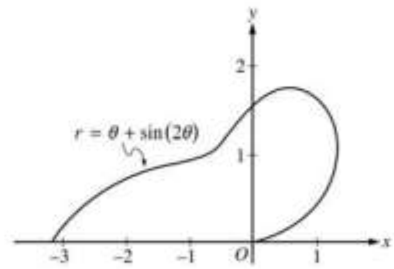
AP Calculus BC
Unit 11 – Parametric Equations & Polar Coordinates

Day 8 Notes: Polar Graphs & Area (Day 2)

2005 BC Exam - #2 – Calculator Active



The curve above is drawn in the *xy*-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

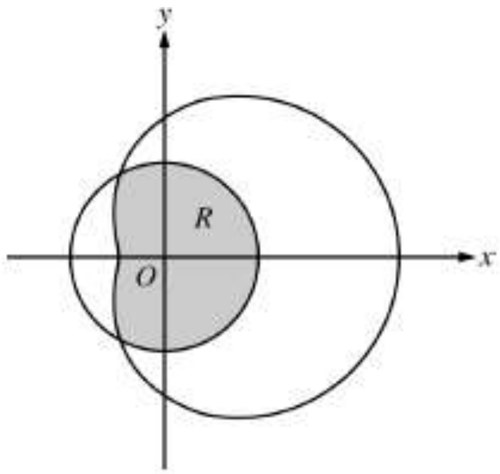
(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

2007 BC Exam - #3 – Calculator Active



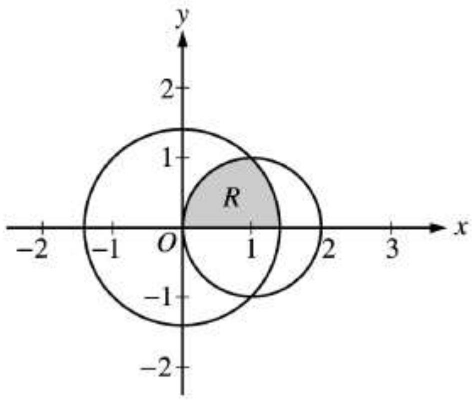
The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

2003 BC Exam (Form B) - #2 – No Calculator



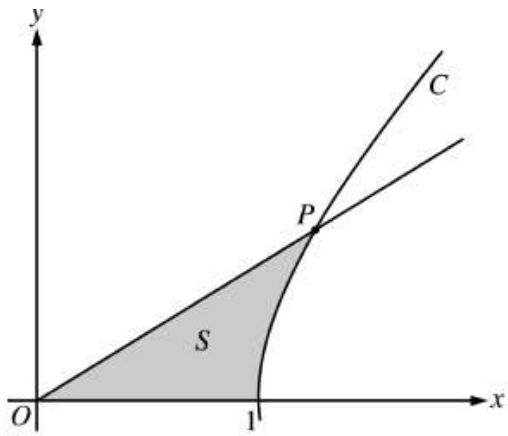
The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and $(x-1)^2 + y^2 = 1$. The graphs intersect at the points $(1, 1)$ and $(1, -1)$. Let R be the shaded region in the first quadrant bounded by the two circles and the x-axis.

(a) Set up an expression involving one or more integrals with respect to x that represents the area of R.

(b) Set up an expression involving one or more integrals with respect to y that represents the area of R.

(c) The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle θ that represents the area of R.

2003 BC Exam - #3 – Calculator Active



The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

(a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.

(b) Set up and evaluate an integral expression with respect to y that gives the area of S.

(c) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$.

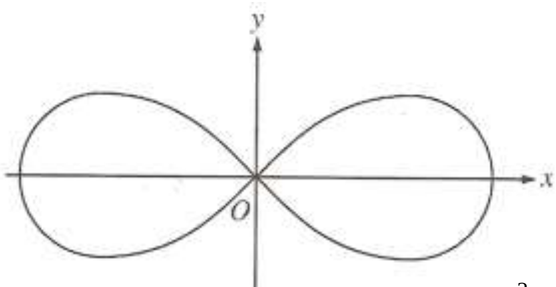
(d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S.

AP Calculus BC **Name: ______________________________**
Unit 11 – Day 8 – Assignment

1. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2 d\theta$ (B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta) d\theta$ (C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2 d\theta$

(D) $\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2 d\theta$ (E) $\int_{7\pi/6}^{-\pi/6} (1+2\sin\theta) d\theta$



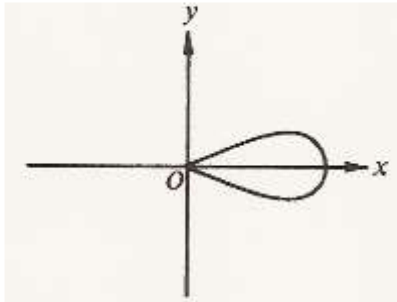
2. What is the area of the region enclosed by the lemniscate $r^2 = 18\cos(2\theta)$ shown in the figure above?

(A) $\frac{9}{2}$ (B) 9 (C) 18 (D) 24 (E) 36

3. The area of one loop of the graph of the polar equation $r = 2\sin(3\theta)$ is given by which of the following expressions?

(A) $4\int_0^{\pi/3} \sin^2(3\theta) d\theta$ (B) $2\int_0^{\pi/3} \sin(3\theta) d\theta$ (C) $2\int_0^{\pi/3} \sin^2(3\theta) d\theta$

(D) $2\int_0^{2\pi/3} \sin^2(3\theta) d\theta$ (E) $2\int_0^{2\pi/3} \sin(3\theta) d\theta$



4. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r = 4\cos(3\theta)$ shown in the figure above?

(A) $16\int_{-\pi/3}^{\pi/3} \cos(3\theta)\,d\theta$ (B) $8\int_{-\pi/6}^{\pi/6} \cos(3\theta)\,d\theta$ (C) $8\int_{-\pi/3}^{\pi/3} \cos^2(3\theta)\,d\theta$

(D) $16\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta$ (E) $8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta$

5. The area of the region enclosed by the polar curve $r = \sin(2\theta)$ for $0 \le \theta \le \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$