AP Calculus BC
Unit 11 – Day 8 – Assignment

Name: Answer Key*

1. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)^2 d\theta$ (B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)\,d\theta$ (C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6}(1+2\sin\theta)^2 d\theta$

(D) $\int_{-\pi/6}^{7\pi/6}(1+2\sin\theta)^2 d\theta$ (E) $\int_{7\pi/6}^{-\pi/6}(1+2\sin\theta)\,d\theta$

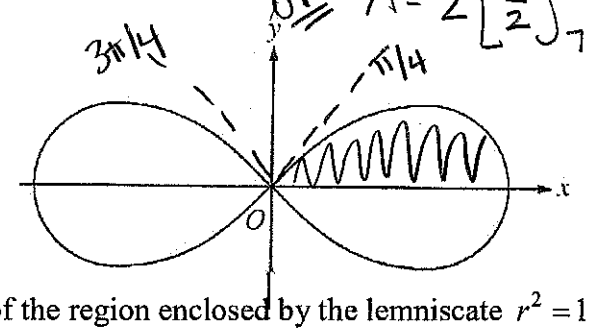
$1 + 2\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

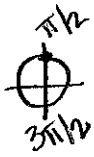
$A = \frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)^2 d\theta = 0.544$

OR $A = 2\left[\frac{1}{2}\int_{7\pi/6}^{9\pi/6}(1+2\sin\theta)^2 d\theta\right] = 0.544$



2. What is the area of the region enclosed by the lemniscate $r^2 = 18\cos(2\theta)$ shown in the figure above?

(A) $\frac{9}{2}$ (B) 9 (C) 18 (D) 24 (E) 36



$18\cos(2\theta) = 0$

$\cos(2\theta) = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{4}, 3\pi/4$

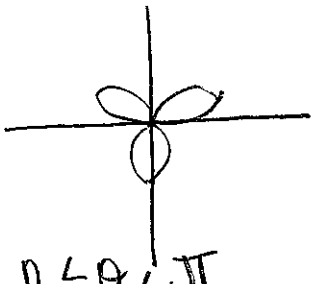
$A = \frac{1}{2}\int_0^{\pi/4}(18\cos 2\theta)\,d\theta = 4.5$

$4.5(4) = 18$

3. The area of one loop of the graph of the polar equation $r = 2\sin(3\theta)$ is given by which of the following expressions?

(A) $4\int_0^{\pi/3}\sin^2(3\theta)\,d\theta$ (B) $2\int_0^{\pi/3}\sin(3\theta)\,d\theta$ (C) $2\int_0^{\pi/3}\sin^2(3\theta)\,d\theta$

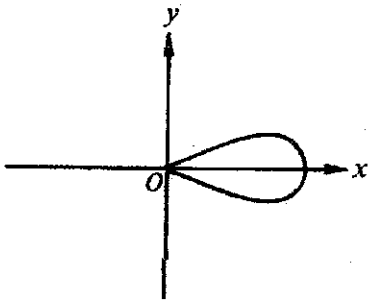
(D) $2\int_0^{2\pi/3}\sin^2(3\theta)\,d\theta$ (E) $2\int_0^{2\pi/3}\sin(3\theta)\,d\theta$



$0 \le \theta \le \pi$

$A = \frac{1}{2}\int_0^{\pi/3}(2\sin 3\theta)^2\,d\theta$

$= \frac{1}{2}\int_0^{\pi/3} 4\sin^2 3\theta\,d\theta = 2\int_0^{\pi/3}\sin^2 3\theta\,d\theta$



4. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r = 4\cos(3\theta)$ shown in the figure above?

(A) $16\int_{-\pi/3}^{\pi/3} \cos(3\theta)\,d\theta$ (B) $8\int_{-\pi/6}^{\pi/6} \cos(3\theta)\,d\theta$ (C) $8\int_{-\pi/3}^{\pi/3} \cos^2(3\theta)\,d\theta = 8.376$

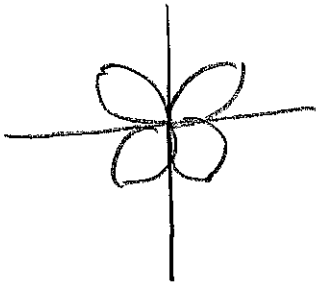
(D) $16\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta$ (E) $8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta = 4.189$

$$A = \frac{1}{2}\int_0^{\pi/2} (4\cos(3\theta))^2\,d\theta = 4.189$$

$$= \frac{1}{2}\int 16\cos^2 3\theta\,d\theta = 8\int \cos^2 3\theta\,d\theta$$

5. The area of the region enclosed by the polar curve $r = \sin(2\theta)$ for $0 \le \theta \le \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$



$$A = \frac{1}{2}\int_0^{\pi/2} \sin^2(2\theta)\,d\theta = 0.393 = \frac{\pi}{8}$$