AP Calculus BC
Unit 11 – Day 7 – Warm-up

Name: Answer Key**

1) A particle follows a path defined parametrically by $x(t) = 2\sqrt{t-3}$ and $y(t) = 3t^2$. What is the speed of the particle at t = 4?

$x(t) = 2(t-3)^{1/2}$
$x'(t) = (t-3)^{-1/2}$ $\quad y'(t) = 6t$

$\text{Speed} = \sqrt{[(4-3)^{-1/2}]^2 + [6(4)]^2}$

$= \boxed{24.021}$

2) An object moving along a curve in the xy-plane has position (x(t), y(t)) at time t with $\frac{dx}{dt} = cos(t^3)$ and $\frac{dy}{dt} = 3sin(t^2)$ for $0 \le t \le 3$. At time t = 2, the object is at position (4, 5). Find the position of the object at time t = 3.

$x(2) = 4 \quad y(2) = 5 \quad x(3) = ? \quad y(3) = ?$

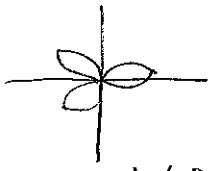
$x(3) - x(2) = \int_2^3 \cos(t^3)\,dt$ $\quad y(3) - y(2) = \int_2^3 3\sin(t^2)\,dt$

$x(3) - 4 = -.046$ $\quad y(3) - 5 = -.094$

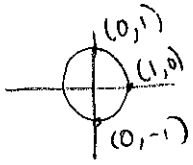
$\boxed{x(3) = 3.954}$ $\quad \boxed{y(3) = 4.906}$

$\boxed{(3.954, 4.906)}$

3) Find the tangents at the pole for the polar curve r = 2cos3θ.



$0 \le \theta < \pi \rightarrow 0 \le 3\theta < 3\pi$



$2\cos 3\theta = 0$
$\cos 3\theta = 0$
$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\frac{\pi}{2} + \frac{4\pi}{2}$

$\boxed{\theta = \frac{\pi}{6}}$ $\quad \theta = \frac{3\pi}{6} = \boxed{\frac{\pi}{2}}$ $\quad \boxed{\theta = \frac{5\pi}{6}}$

4) Find the perimeter of the curve r = 4(1 – sinθ).

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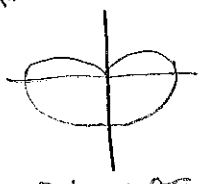


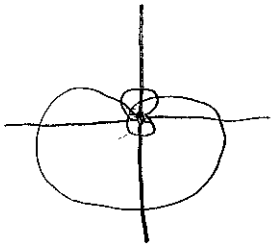
$0 \le \theta < 2\pi$

$r = 4 - 4\sin\theta \rightarrow r'(t) = -4\cos\theta$

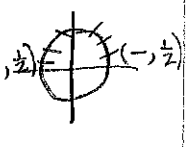
$\int_0^{2\pi} \sqrt{(4-4\sin\theta)^2 + (-4\cos\theta)^2}\,d\theta = \boxed{32}$

5) Find the points of intersection for r = 2 – 3sinθ and r = sinθ.

$\boxed{\text{intersects at pole} \rightarrow (0,0)}$



$2 - 3\sin\theta = \sin\theta$
$2 = 4\sin\theta$
$\frac{1}{2} = \sin\theta$
$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$r = \sin(\frac{\pi}{6}) = \frac{1}{2}$ $\quad r = \sin(\frac{5\pi}{6}) = \frac{1}{2}$

$\boxed{(\frac{1}{2}, \pi/6)}$ $\quad \boxed{(\frac{1}{2}, \frac{5\pi}{6})}$