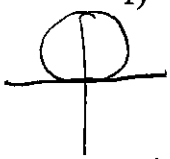
AP Calculus BC
Unit 11 – Day 7 – Assignment

Name: Answer Key*

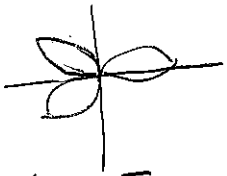
1) Find the area of the region bounded by the graph of $r = 8\sin\theta$.



$0 \le \theta < \pi$

$A = \frac{1}{2}\int_0^{\pi} (8\sin\theta)^2 \, d\theta = \boxed{50.265}$

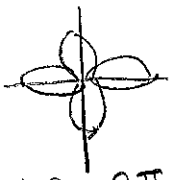
2) Find the area of one petal of $4 = 2\cos 3\theta$.



$0 \le \theta < \pi$

$A = \frac{1}{2}\int_0^{\pi/3} (2\cos 3\theta)^2 \, d\theta = \boxed{1.047}$

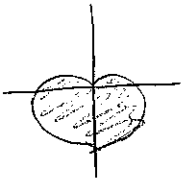
3) Find the area of one petal of $r = \cos 2\theta$.



$0 \le \theta < 2\pi$

$A = \frac{1}{2}\int_0^{\pi/2} (\cos 2\theta)^2 \, d\theta = \boxed{0.393}$

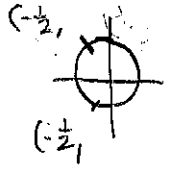
4) Find the area of the interior of $r = 1 - \sin\theta$.



$0 \le \theta < 2\pi$

$A = \frac{1}{2}\int_0^{2\pi} (1-\sin\theta)^2 \, d\theta = \boxed{4.712}$

5) Find the area of the inner loop of $r = 1 + 2\cos\theta$.

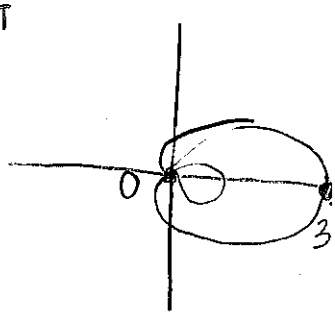


$0 = 1 + 2\cos\theta$

$-\frac{1}{2} = \cos\theta$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ ← halfway is $\frac{3\pi}{3} = \pi$

$A = 2\left[\frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi} (1+2\cos\theta)^2 \, d\theta\right] = \boxed{0.544}$



6) Find the area of the region between the loops of $r = 1 + 2\cos\theta$.

outer loop: $0 = 1 + 2\cos\theta$ $\quad 3 = 1 + 2\cos\theta$

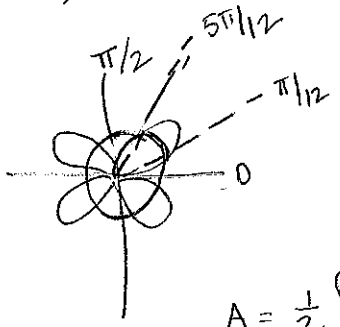
$\theta = 2\pi/3, 4\pi/3$ $\quad 1 = \cos\theta$

$\theta = 0$

$A = 2\left[\frac{1}{2}\int_0^{2\pi/3} (1+2\cos\theta)^2 \, d\theta\right] = 8.881$

Area B/t LOOPS $= 8.881 - 0.544 = \boxed{8.337}$

7) Find the area of the common interior region of $r = 4\sin 2\theta$ and $r = 2$.



$4\sin 2\theta = 2$

$\sin 2\theta = 1/2$

$2\theta = \pi/6, 5\pi/6$

$\theta = \pi/12, 5\pi/12$

$A = \frac{1}{2}\int_0^{\pi/12} (4\sin 2\theta)^2 d\theta = 0.181$

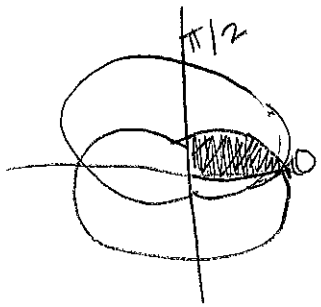
$A = \frac{1}{2}\int_{\pi/12}^{5\pi/12} (2)^2 d\theta = 2.094$

$A = \frac{1}{2}\int_{5\pi/12}^{\pi/2} (4\sin 2\theta)^2 d\theta = 0.181$

$0.181 + 2.094 + .181 = 2.456$

$2.456(4) = \boxed{9.824}$

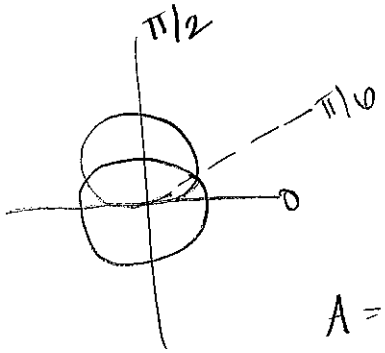
8) Find the area of the common interior region of $r = 3 - 2\sin\theta$ and $r = -3 + 2\sin\theta$.



$A = \frac{1}{2}\int_0^{\pi/2} (3 - 2\sin\theta)^2 d\theta = 2.639$

$2.639(4) = \boxed{10.556}$

9) Find the area of the common interior region of $r = 4\sin\theta$ and $r = 2$.



$4\sin\theta = 2$

$\sin\theta = 1/2$

$\theta = \pi/6, 5\pi/6$

$A = \frac{1}{2}\int_0^{\pi/6} (4\sin\theta)^2 d\theta = 0.362$

$A = \frac{1}{2}\int_{\pi/6}^{\pi/2} (2)^2 d\theta = 2.094$

$0.362 + 2.094 = 2.456$

$2.456(2) = \boxed{4.912}$