AP Calculus BC
Unit 11 – Parametric Equations & Polar Coordinates

Day 6 Notes: Polar Graphs & Arc Length

ARC LENGTH: Let s be the length of an arc of a curve. You must be able to find s for equations in rectangular, parametric, and polar forms.

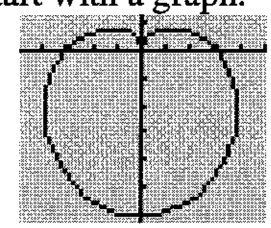
Rectangular: $$s=\int_a^b \sqrt{1+\left(f'(x)\right)^2}\,dx$$

Parametric: $$s=\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$$

Polar: $$s=\int_{\theta_1}^{\theta_2} \sqrt{\left(r(\theta)\right)^2+\left(r'(\theta)\right)^2}\,d\theta$$

Example #1: Find the perimeter of the curve $r=3(1-\sin\theta)$. $r=3-3\sin\theta$

Start with a graph:



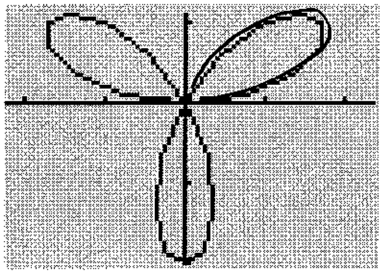
$0 \le \theta < 2\pi$

Strategy:
1. ~~Find the arc length of half the curve and double it.~~
2. ~~Find the interval that traces out half the curve.~~

$$\int_0^{2\pi} \sqrt{(3-3\sin\theta)^2+(-3\cos\theta)^2}\,d\theta$$

$$=\boxed{24}$$

Example #2: Find the perimeter of one petal of the rose curve $r=2\sin(3\theta)$.



$$\int_0^{\pi/3} \sqrt{(2\sin 3\theta)^2+(6\cos(3\theta))^2}\,d\theta = \boxed{4.455}$$

$0 \le \theta < \pi \rightarrow$ one petal: $0 \le \theta < \pi/3$

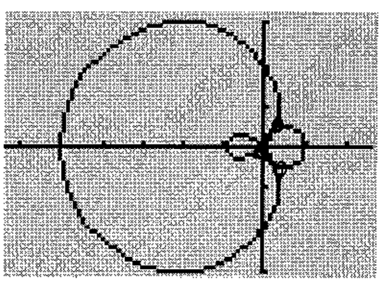
POINTS OF INTERSECTION OF POLAR GRAPHS

To find points of intersection, begin by graphing. Then solve the polar equations simultaneously. Sometimes, not all of the points of intersection will show up with the algebraic solution. Usually the missing points are at the pole.

- The points of intersection that you find algebraically are reached simultaneously (for the same values of Θ) in both graphs.
- The graphs may reach the pole at different values of Θ, which is why these solutions don't show up in the algebraic solution.

Note: When you are asked to find points of intersection, your answers will be in the form (r, Θ).

Example #3: Find the points of intersection of the graphs of $r = 2 - 3\cos\theta$ and $r = \cos\theta$.



3 pts.

$2 - 3\cos\theta = \cos\theta$

$2 = 4\cos\theta$

$\frac{1}{2} = \cos\theta$

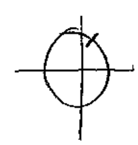
$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$r = \cos(\pi/3) = \frac{1}{2}$

$r = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$

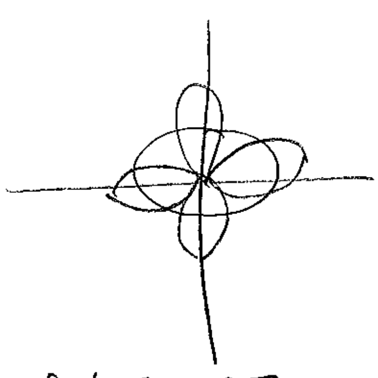
$\boxed{\begin{array}{c} (\frac{1}{2}, \pi/3) \\ (\frac{1}{2}, 5\pi/3) \\ (0, 0) \end{array}}$ ← intersects at the pole

☆ *Example #4*: Find the points of intersection of the graphs of $r = 2$ and $r = 4\cos(2\theta)$.



$4\cos(2\theta) = \pm 2$

$\cos(2\theta) = \pm\frac{1}{2}$



$2\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6}$

$\frac{\pi}{3}$ $\frac{2\pi}{3}$ $\frac{4\pi}{3}$ $\frac{5\pi}{3}$

$0 \le \theta < 2\pi$

$0 \le 2\theta < 4\pi$

$(2, \pi/6), (2, \pi/3), (2, 2\pi/3), (2, 5\pi/6)$
$(2, 7\pi/6), (2, 4\pi/3)\ (2, 5\pi/3)\ (2, 11\pi/6)$