AP Calculus BC
Unit 11 – Day 5 – Assignment

Name: Answer Key*

1) Find dy/dx for $r = 2 + 3\sin\theta$.

$$\frac{dy}{dx} = \frac{2\cos\theta + 6\sin\theta\cos\theta}{-2\sin\theta + 3\cos^2\theta - 3\sin^2\theta}$$

2) Find dy/dx for $r = 3(1 - \cos\theta)$ at $\theta = \pi/2$.

-1

3) Find dy/dx for $r = 3\sin\theta$ at $\theta = \pi/3$.

$-\sqrt{3}$

4) Find the points of horizontal and vertical tangency to the polar curve $r = 1 - \sin\theta$.

horizontal: $(2, 3\pi/2)$, $(\frac{1}{2}, \pi/6)$, $(\frac{1}{2}, \frac{5\pi}{6})$

vertical: $(\frac{3}{2}, \frac{7\pi}{6})$, $(\frac{3}{2}, \frac{11\pi}{6})$

5) **Find the points of horizontal tangency to the polar curve $r = 2\csc\theta + 3$.**

$(5, \pi/2)$

$(1, 3\pi/2)$

6) **Sketch the graph of the polar equation and find the tangents at the pole for the polar curve $r = 2(1 - \sin\theta)$.**

$\pi/2$

7) **Sketch the graph of the polar equation and find the tangents at the pole for the polar curve $r = 2\cos3\theta$.**

$\pi/6, \pi/2, 5\pi/6$

8) **Sketch the graph of the polar equation and find the tangents at the pole for the polar curve $r = 3\sin2\theta$.**

$0, \pi/2, \pi, 3\pi/2$

Unit 11-Day 5 Assignment

① $r = 2 + 3\sin\theta$

$x = (2 + 3\sin\theta)(\cos\theta)$ $\quad y = (2 + 3\sin\theta)(\sin\theta)$

$x = 2\cos\theta + 3\sin\theta\cos\theta$ $\quad y = 2\sin\theta + 3\sin^2\theta$

$$\frac{dy}{dx} = \frac{2\cos\theta + 6(\sin\theta)(\cos\theta)}{-2\sin\theta + 3\cos\theta\cos\theta + 3\sin\theta(-\sin\theta)}$$

$$\boxed{\frac{dy}{dx} = \frac{2\cos\theta + 6\sin\theta\cos\theta}{-2\sin\theta + 3\cos^2\theta - 3\sin^2\theta}}$$

② $r = 3(1 - \cos\theta)$, $\theta = \pi/2$

$r = 3 - 3\cos\theta$

$x = (3 - 3\cos\theta)(\cos\theta)$ $\quad y = (3 - 3\cos\theta)(\sin\theta)$

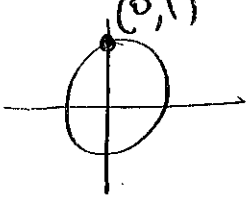
$x = 3\cos\theta - 3\cos^2\theta$ $\quad y = 3\sin\theta - 3\cos\theta\sin\theta$

$$\frac{dy}{dx} = \frac{3\cos\theta + 3\sin\theta\sin\theta - 3\cos\theta\cos\theta}{-3\sin\theta + 6\cos\theta\sin\theta}$$

$$= \frac{3\cos\theta + 3\sin^2\theta - 3\cos^2\theta}{-3\sin\theta + 6\cos\theta\sin\theta}$$

at $\theta = \pi/2$

$$= \frac{0 + 3 - 0}{-3 + 0} = \frac{3}{-3} = \boxed{-1}$$



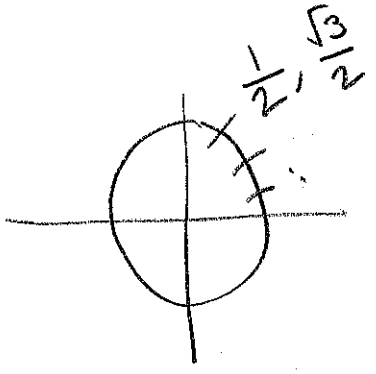
(3) $r = 3\sin\theta$, $\theta = \pi/3$

$X = (3\sin\theta)(\cos\theta)$ $\quad Y = (3\sin\theta)(\sin\theta)$

$X = 3\sin\theta\cos\theta$ $\quad Y = 3\sin^2\theta$

$$\frac{dy}{dx} = \frac{6\sin\theta\cos\theta}{3\cos\theta\cos\theta + 3\sin\theta(-\sin\theta)}$$

$$= \frac{6\sin\theta\cos\theta}{3\cos^2\theta - 3\sin^2\theta}$$



at $\theta = \pi/3$

$$= \frac{6\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{3\left(\frac{1}{2}\right)^2 - 3\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{6\sqrt{3}}{4}}{3\left(\frac{1}{4}\right) - 3\left(\frac{3}{4}\right)} = \frac{\frac{3\sqrt{3}}{2}}{\frac{3}{4} - \frac{9}{4}} = \frac{\frac{3\sqrt{3}}{2}}{\frac{-6}{4}} = \frac{3\sqrt{3}}{2} \cdot \frac{4}{-6}$$

$$= \frac{12\sqrt{3}}{-12} = \boxed{-\sqrt{3}}$$

(4) $r = 1 - \sin\theta$

$x = (1-\sin\theta)(\cos\theta)$ $\quad y = (1-\sin\theta)\sin\theta)$

$x = \cos\theta - \sin\theta\cos\theta$ $\quad y = \sin\theta - \sin^2\theta$

horizontal: $\frac{dy}{d\theta} = 0$ $\quad \cos\theta - 2\sin\theta\cos\theta = 0$

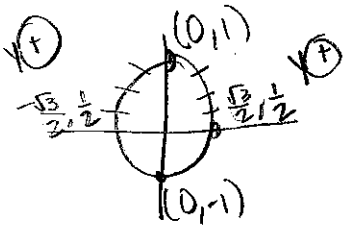
$\cos\theta(1 - 2\sin\theta) = 0$

$\cos\theta = 0$ $\quad 1 - 2\sin\theta = 0$

$\theta = \cancel{\frac{\pi}{2}}, \frac{3\pi}{2}$ $\quad 1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$r = 1 - \sin(\frac{3\pi}{2})$

$r = 1 - (-1)$ $\quad r = 2$

$(2, 3\pi/2)$

$r = 1 - \sin(\pi/6) = 1 - (\frac{1}{2}) = \frac{1}{2}$

$r = 1 - \sin(\frac{5\pi}{6}) = 1 - (\frac{1}{2}) = \frac{1}{2}$

$(\frac{1}{2}, \frac{\pi}{6})$

$(\frac{1}{2}, \frac{5\pi}{6})$

Vertical: $\frac{dx}{d\theta} = 0$ $\quad -\sin\theta - \cos\theta\cos\theta - \sin\theta(-\sin\theta) = 0$

$-\sin\theta - \cos^2\theta + \sin^2\theta = 0$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$-\sin\theta - (1-\sin^2\theta) + \sin^2\theta = 0$

$-\sin\theta - 1 + \sin^2\theta + \sin^2\theta = 0$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin^2\theta - 2\sin\theta) + 1\sin\theta - 1 = 0$

$2\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -1/2$ $\quad \sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad \theta = \cancel{\pi/2}$

$r = 1 - \sin(\frac{7\pi}{6})$

$r = 1 - (-\frac{1}{2})$

$r = 3/2$

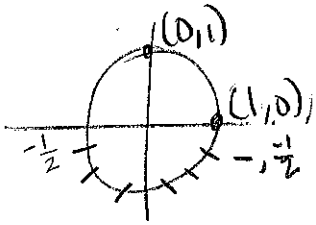
$r = 1 - \sin(\frac{11\pi}{6})$

$r = 1 - (-\frac{1}{2})$

$(\frac{3}{2}, \frac{7\pi}{6})$

$(\frac{3}{2}, \frac{11\pi}{6})$

$\begin{array}{r|l} -2 & \\ \hline -2 & 1 \end{array}$



(5) $r = 2\csc\theta + 3$

$x = (2\csc\theta + 3)(\cos\theta)$ $\quad y = (2\csc\theta + 3)(\sin\theta)$

$y = 2\csc\theta\sin\theta + 3\sin\theta$

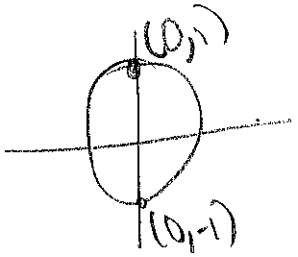
$y = 2 + 3\sin\theta$

horizontal: $\frac{dy}{d\theta} = 0$

$3\cos\theta = 0$

$\cos\theta = 0$

$\theta = \pi/2, 3\pi/2$

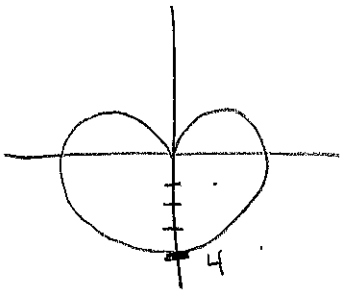


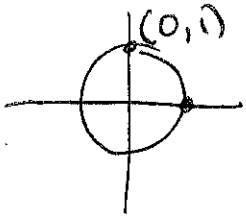
$r = 2\csc(\frac{\pi}{2}) + 3 = 2(1) + 3 = 5$ $\quad \boxed{(5, \frac{\pi}{2})}$

$r = 2\csc(\frac{3\pi}{2}) + 3 = 2(-1) + 3 = 1$ $\quad \boxed{(1, \frac{3\pi}{2})}$

(6) $r = 2(1 - \sin\theta)$

$r = 2 - 2\sin\theta$ cardioid





$2 - 2\sin\theta = 0$

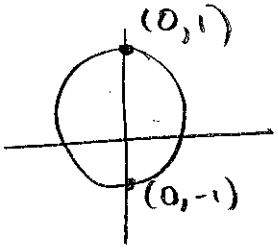
$-2\sin\theta = -2$

$\sin\theta = 1$

$\boxed{\theta = \pi/2}$ $\quad 0 \le \theta < 2\pi$

(7) $r = 2\cos 3\theta$ $\quad 0 \le \theta < \pi$

$0 \le 3\theta < 3\pi$



$2\cos 3\theta = 0$

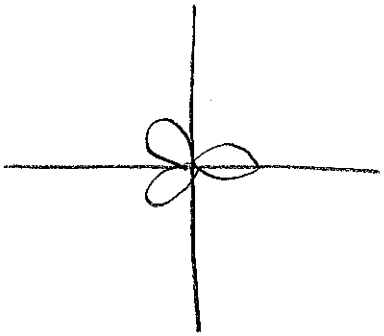
$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$3\theta = \frac{\pi}{2}$ $\quad 3\theta = \frac{3\pi}{2}$ $\quad 3\theta = \frac{5\pi}{2}$

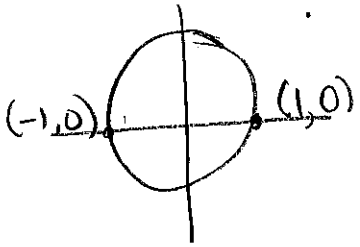
$\theta = \frac{\pi}{6}$ $\quad \theta = \frac{3\pi}{6}$ $\quad \theta = \frac{5\pi}{6}$

$\theta = \pi/2$



(8) $r = 3\sin 2\theta$ $\quad 0 \le \theta < 2\pi$

$0 \le 2\theta < 4\pi$



$3\sin 2\theta = 0$

$\sin 2\theta = 0$

$2\theta = 0, \pi, 2\pi, 3\pi$

$2\theta = 0$ $\quad 2\theta = \pi$ $\quad 2\theta = 2\pi$ $\quad 2\theta = 3\pi$

$\theta = 0$ $\quad \theta = \frac{\pi}{2}$ $\quad \theta = \pi$ $\quad \theta = \frac{3\pi}{2}$