AP Calculus BC
Unit 11 – Day 4 – Warm-up

Name: Answer Key*

AP® CALCULUS BC
2006 SCORING GUIDELINES (Form B)

Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

(a) Write an equation for the line tangent to the curve at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.

(d) Is there a time $t \geq 0$ at which the object is on the y-axis? Explain why or why not.

(a) $\text{slope} = \dfrac{dy}{dx} = \dfrac{\sec(e^{-t})}{\tan(e^{-t})} = \dfrac{\sec(e^{-1})}{\tan(e^{-1})} = 2.781$

$t=1$ at $(2,-3)$

$$\boxed{y+3 = 2.781(x-2)}$$

(b) $\langle x''(1), y''(1) \rangle = \boxed{\langle -.423, -.152 \rangle}$

$\text{Speed} = \sqrt{(\tan(e^{-1}))^2 + (\sec(e^{-1}))^2} = \boxed{1.139}$

(c) $\displaystyle\int_1^2 \sqrt{(\tan(e^{-t}))^2 + (\sec(e^{-t}))^2}\, dt = \boxed{1.059}$

(d) $x(0) = ?$, $x(1) = 2$

$x(1) - x(0) = \int_0^1 \tan(e^{-t})\, dt$

$2 - x(0) = 0.776$

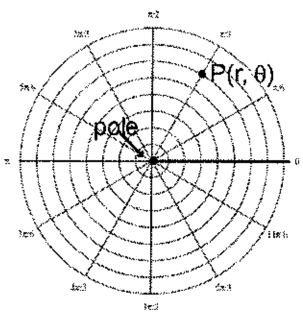
$x(0) = 1.224 > 0$

The particle starts to the Right of the y-axis. Since $x'(t) > 0$ for all $t \geq 0$ the object is always moving Right. $\therefore$ Never on the y-axis

AP Calculus BC

Unit 11 – Parametric Equations & Polar Coordinates

Day 4: Polar Coordinates



Polar coordinates for a point P are in the form P(r, θ), where r is the distance from the origin (pole) to P and θ is the angle from the polar axis to OP. θ is positive when measured in the counterclockwise direction and negative in the clockwise direction.

With rectangular coordinates, each point (x, y) is a unique point. Not so with polar coordinates.

**The pole can be written as (0, θ), where θ is any angle.

Pole : $(0, \theta)$

$\frac{5\pi}{4} + \frac{8\pi}{4} = \frac{13\pi}{4}$

Plot these points:

A(2, 5π/4)

$(-2, \pi/4)$

$(2, \frac{13\pi}{4})$

B(0, -7π/6)

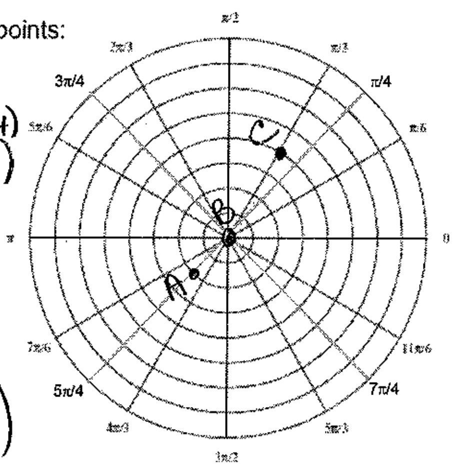
$(0, 0)$

$(0, \pi/2)$

C(-4, 4π/3)

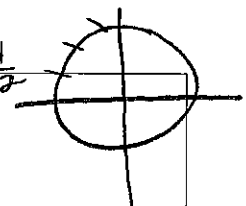
$(4, \pi/3)$

$(-4, \frac{10\pi}{3})$



Give 2 other names for each point.

$\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$



$-\frac{\sqrt{3}}{2}, \frac{1}{2}$

CONVERT COORDINATES:

Polar to rectangular: (r, θ) → (x, y)

Use x = r cosθ and y = r sinθ.

Convert (-2, 5π/6) into rectangular coordinates.

r θ

$x = r\cos\theta$

$x = -2\cos(\frac{5\pi}{6})$

$-2(-\frac{\sqrt{3}}{2})$

$x = \sqrt{3}$

$y = r\sin\theta$

$y = -2\sin(\frac{5\pi}{6})$

$y = -2(\frac{1}{2})$

$y = -1/2$

$\boxed{(\sqrt{3}, -\frac{1}{2})}$

Rectangular to Polar: (x, y) → (r, θ)

Use $r^2 = x^2 + y^2$ and $\tan\theta = y/x$.

Find two sets of polar coordinates for (3, -1).

x y

$r^2 = x^2 + y^2$

$r^2 = (3)^2 + (-1)^2$

$r^2 = 9 + 1$

$r = \pm\sqrt{10}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-1}{3}$

$\theta = -.322$

$(\sqrt{10}, -.322)$

$(-\sqrt{10}, \pi - .322)$

CONVERTING EQUATIONS

x = r cosθ
y = r sinθ
$r^2 = x^2 + y^2$
tanθ = y/x

Polar to rectangular:

1) r = - 2

$r^2 = x^2 + y^2$

$(-2)^2 = x^2 + y^2$

$\boxed{x^2 + y^2 = 4}$

2) r = 3 cosθ

$r = 3(\frac{x}{r})$

$r^2 = 3x$

$x^2 + y^2 = 3x$

$x^2 - 3x + (\frac{3}{2})^2 + y^2 = (\frac{3}{2})^2$

$\boxed{(x - \frac{3}{2})^2 + y^2 = \frac{9}{4}}$

3) θ = 5π/6

$\tan\theta = \frac{y}{x}$

$\tan(\frac{5\pi}{6}) = \frac{y}{x}$

$\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{y}{x}$

$\frac{1}{-\sqrt{3}} = \frac{y}{x}$

$x = -\sqrt{3}y$

$\boxed{\frac{x}{-\sqrt{3}} = y}$

Rectangular to polar:

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2 + y^2$
$\tan\theta = y/x$

1) $x^2 + y^2 = 2x$

$r^2 = 2r\cos\theta$

$\boxed{r = 2\cos\theta}$

2) $x = 10$

$r\cos\theta = 10$ $\quad r = \dfrac{10}{\cos\theta}$ $\quad \boxed{r = 10\sec\theta}$

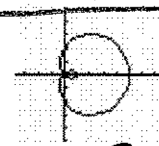
3) $xy = 4$

$(r\cos\theta)(r\sin\theta) = 4$

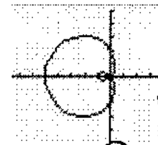
$r^2\cos\theta\sin\theta = 4$ $\quad \boxed{r^2 = \dfrac{4}{\cos\theta\sin\theta}}$

Limacons take the form $r = a \pm b \cdot \cos\theta$ or $r = a \pm b \cdot \sin\theta$

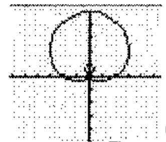
- If a < b, the limaçon has a loop.


$r = 2 + 3\cos\theta$

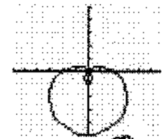
Right


$r = 2 - 3\cos\theta$

Left


$r = 2 + 3\sin\theta$

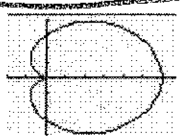
above

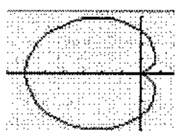

$r = 2 - 3\sin\theta$

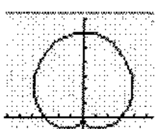
below

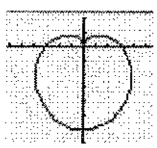
$\boxed{0 \le \theta < 2\pi}$

If a = b, the limacon is called a ~~cardioid~~.

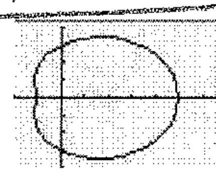

$r = 3 + 3\cos\theta$

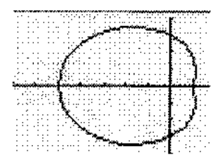

$r = 3 - 3\cos\theta$

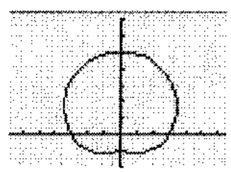

$r = 3 + 3\sin\theta$

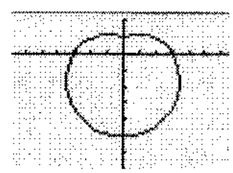

$r = 3 - 3\sin\theta$

If a > b, then we have a dimpled limacon

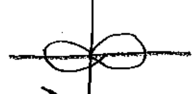
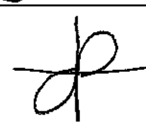

$r = 3 + 2\cos\theta$


$r = 3 - 2\cos\theta$


$r = 3 + 2\sin\theta$


$r = 3 - 2\sin\theta$

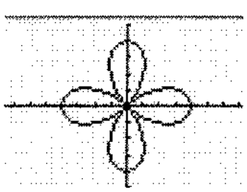
Lemniscate $\quad r^2 = a^2\sin 2\theta$
$\quad r^2 = a^2\cos 2\theta$

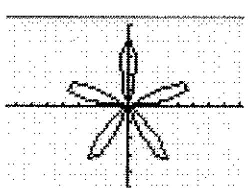



$\boxed{0 \le \theta < 2\pi}$

Rose curves take the form $r = a\cos(n\theta)$ or $r = a\sin(n\theta)$.

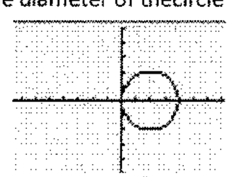
If n is odd, the curve will have n petals.
If n is even, the curve will have 2n petals.
The length of each petal is a.
Curves in the form $r = a\cos(n\theta)$ will have a petal along the polar axis.

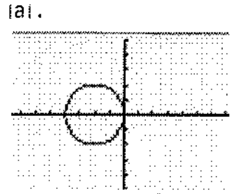

$r = 4\cos(2\theta)$

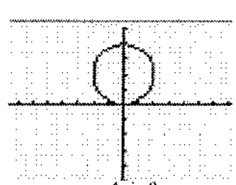

$r = 4\sin(5\theta)$

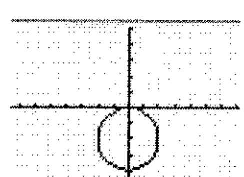
Traced only once: $0 \le \theta < 2\pi$

Circles can also have polar form $r = a\cos\theta$ or $r = a\sin\theta$.
The diameter of thecircle will be $|a|$.


$r = 4\cos\theta$


$r = -4\cos\theta$


$r = 4\sin\theta$


$r = -4\sin\theta$

Traced only once: $0 \le \theta < \pi$