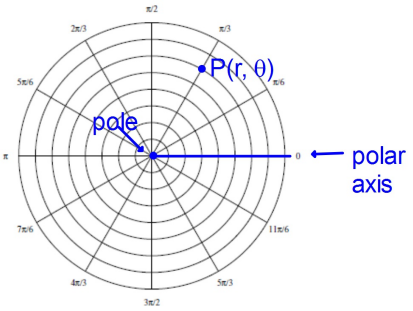
AP Calculus BC

Unit 11 – Parametric Equations & Polar Coordinates

Day 4: Polar Coordinates



Polar coordinates for a point P are in the form P(r, θ), where r is the distance from the origin (pole) to P and θ is the angle from the polar axis to OP. θ is positive when measured in the counterclockwise direction and negative in the clockwise direction.

With rectangular coordinates, each point (x, y) is a unique point. Not so with polar coordinates.

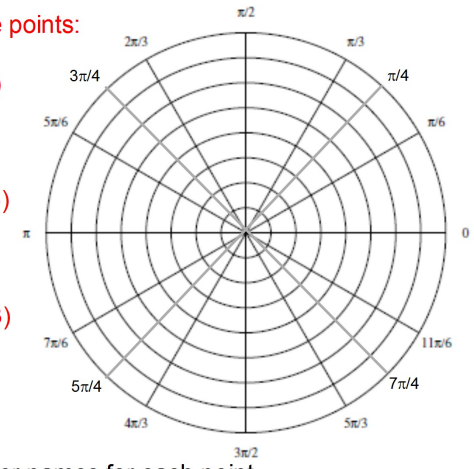
**The pole can be written as (0, θ), where θ is any angle.

Plot these points:

A(2, 5π/4)

B(0, -7π/6)

C(-4, 4π/3)



Give 2 other names for each point.

CONVERT COORDINATES:

Polar to rectangular: (r, θ) → (x, y)

Use $x = r\cos\theta$ and $y = r\sin\theta$.

Convert (-2, 5π/6) into rectangular coordinates.

Rectangular to Polar: (x, y) → (r, θ)

Use $r^2 = x^2 + y^2$ and $\tan\theta = y/x$.

Find two sets of polar coordinates for (3, -1).

CONVERTING EQUATIONS

Polar to rectangular:

1) r = - 2

2) r = 3 cosθ

3) θ = 5π/6

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2 + y^2$
$\tan\theta = y/x$

Rectangular to polar:

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2 + y^2$
$\tan\theta = y/x$

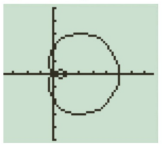
1) $x^2 + y^2 = 2x$

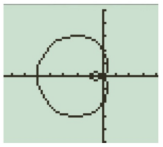
2) $x = 10$

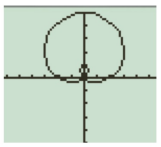
3) $xy = 4$

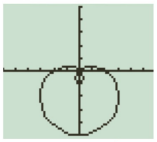
Limacons take the form $r = a \pm b\cdot\cos\theta$ or $r = a \pm b\cdot\sin\theta$

If a < b, the limaçon has a loop.

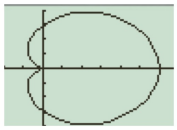

$r = 2 + 3\cos\theta$

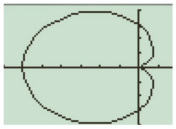

$r = 2 - 3\cos\theta$

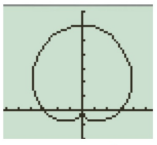

$r = 2 + 3\sin\theta$

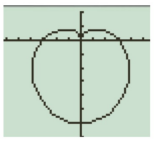

$r = 2 - 3\sin\theta$

If a = b, the limacon is called a cardioid.

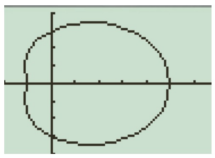

$r = 3 + 3\cos\theta$

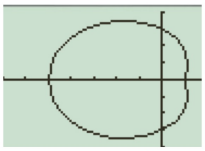

$r = 3 - 3\cos\theta$

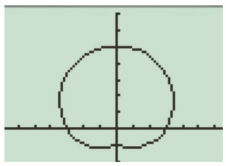

$r = 3 + 3\sin\theta$

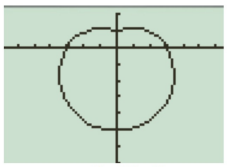

$r = 3 - 3\sin\theta$

If a > b, then we have a dimpled limacon.


$r = 3 + 2\cos\theta$

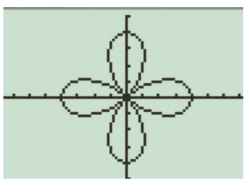

$r = 3 - 2\cos\theta$

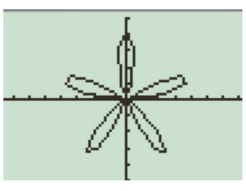

$r = 3 + 2\sin\theta$


$r = 3 - 2\sin\theta$

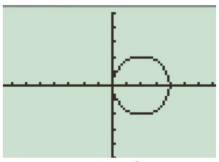
Rose curves take the form $r = a\cos(n\theta)$ or $r = a\sin(n\theta)$.

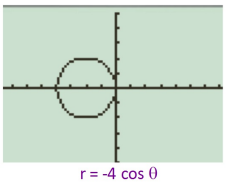
If n is odd, the curve will have n petals.
If n is even, the curve will have 2n petals.
The length of each petal is a.
Curves in the form $r = a\cos(n\theta)$ will have a petal along the polar axis.

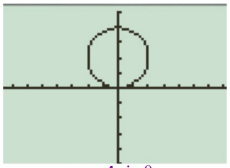

$r = 4\cos(2\theta)$

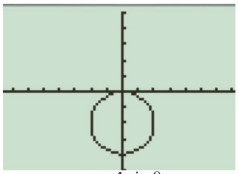

$r = 4\sin(5\theta)$

Circles can also have polar form $r = a\cos\theta$ or $r = a\sin\theta$.
The diameter of thecircle will be $|a|$.


$r = 4\cos\theta$


$r = -4\cos\theta$


$r = 4\sin\theta$


$r = -4\sin\theta$