calc. Active

(1) $x'(t) = \frac{\sqrt{t+2}}{e^t}$, $y'(t) = \sin^2 t$, $x(2) = 1$, $y(2) = 5$

(a) $x'(2) = \frac{\sqrt{2+2}}{e^2} = \frac{2}{e^2} > 0$, then particle is moving to the $\boxed{\text{Right}}$

$\frac{dy}{dx} = \frac{\sin^2 t}{\frac{\sqrt{t+2}}{e^t}}$ → slope at $t=2$ = $\frac{\sin^2(2)}{\frac{\sqrt{2+2}}{e^2}} = \boxed{3.055}$

(b) $x(4) - x(2) = \underbrace{\int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt}$

$x(4) - 1 = 0.253$

$\boxed{x(4) = 1.253}$

(c) speed $= \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + \left(\sin^2 t\right)^2}$

at $t=4$ → $\sqrt{\left(\frac{\sqrt{4+2}}{e^4}\right)^2 + \left(\sin^2 4\right)^2} = \boxed{0.575}$

acceleration vector → $\langle x''(4), y''(4) \rangle$

$\boxed{\langle -.041, 0.989 \rangle}$

(d) $\int_2^4 \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + \left(\sin^2 t\right)^2}\, dt = \boxed{0.651}$

CALC ACTIVE

2. $x' = 4t+1 \quad y' = \sin(t^2) \quad x(0) = 0 \quad y(0) = -4$

a. speed $= \sqrt{(4t+1)^2 + (\sin t^2)^2}$

at $t = 3$ $\sqrt{[4(3)+1]^2 + [\sin(3^2)]^2} = \boxed{13.007}$

acceleration vector $\rightarrow$ $\langle x''(3), y''(3) \rangle$

$\boxed{\langle 4, -5.467 \rangle}$

b. slope $= \frac{dy}{dx} = \frac{\sin(t^2)}{4t+1} = \frac{\sin(3^2)}{4(3)+1} = \boxed{0.032}$

c. $x(3) - x(0) = \int_0^3 4t+1 \, dt$

$x(3) - 0 = 21$

$x(3) = 21$

$y(3) - y(0) = \int_0^3 \sin(t^2) \, dt$

$y(3) - (-4) = 0.774$

$y(3) = -3.226$

$\boxed{(21, -3.226)}$

(d) $\int_0^3 \sqrt{(4t+1)^2 + (\sin(t^2))^2}\, dt = \boxed{21.091}$

CALC ACTIVE

(3) $x(t) = t^2 - 4t + 8 \qquad y(t) = te^{t-3} - 1$

$x'(t) = 2t - 4$

(a) Speed $= \sqrt{(2t-4)^2 + (te^{t-3}-1)^2}$

at $t=3$ $\quad \sqrt{(2)^2 + (3e^0 - 1)^2} = \boxed{2.828 \text{ m/sec}}$

(b) Total Distance $= \int_0^4 \sqrt{(2t-4)^2 + (te^{t-3}-1)^2}\, dt$

$= \boxed{11.588 \text{ m}}$

(c) horizontal tangent when $\frac{dy}{dt} = 0$

$\underbrace{te^{t-3} - 1}_{y_1} = \underbrace{0}_{y_2} \rightarrow \boxed{t = 2.208 \text{ sec}}$

$x'(2.208) = 2(2.208) - 4 = 0.416 > 0$

Since $x'(2.208)$ is positive, the particle moves to the Right.

d) $x(t) = 5$ twice

(i) $x(t) = t^2 - 4t + 8$

$t^2 - 4t + 8 = 5$

$t^2 - 4t + 3 = 0$

$(t+3)(t-1)$

$\boxed{t = 3, \quad t = 1}$

(ii) $\dfrac{dy}{dx} = \dfrac{te^{t-3} - 1}{2t - 4}$

$\underline{t=1}$: $\dfrac{(1)e^{1-3} - 1}{2(1) - 4} = \boxed{0.432}$

$\underline{t=3}$: $\dfrac{(3)e^{3-3} - 1}{2(3) - 4} = \boxed{1}$

SLOPES

(iii) $y(2) = 3 + \frac{1}{e}$, $y(1)$ or $y(3)$?

$y(2) - y(1) = \underbrace{\int_1^2 te^{t-3} - 1\, dt}$

$3 + \frac{1}{e} - y(1) = -.632$

$-y(1) = -4.000$

$\boxed{y(1) = 4.000}$

OR

$y(3) - y(2) = \int_2^3 te^{t-3} - 1\, dt$

$y(3) - (3 + \frac{1}{e}) = 0.632$

$\boxed{y(3) = 4}$

(4) $x'(t) = 14\cos(t^2)\sin(e^t)$ $\quad y'(t) = 1 + 2\sin(t^2)$

$x(0) = -2 \quad y(0) = 3$

(a) vertical tangent → $x'(t) = 0$

$$\underbrace{14\cos(t^2)\sin(e^t)}_{y_1} = \underbrace{0}_{y_2}$$

$\underbrace{0 < t < 1.5}$

Fix Xmin & Xmax

$\boxed{t = 1.145}$ $\boxed{t = 1.253}$

(b) $t = 1$ → equation of tangent line

$$\frac{dy}{dx} = \frac{1 + 2\sin(t^2)}{14\cos(t^2)\sin(e^t)} = \frac{1 + 2\sin(1^2)}{14\cos(1^2)\sin(e^1)} = 0.863 \text{ slope}$$

point: $x(1) = ?$, $y(1) = ?$

$$x(1) - x(0) = \underbrace{\int_0^1 14\cos(t^2)\sin(e^t)\,dt}$$

$x(1) - (-2) = 11.315$

$x(1) = 9.314$

$$y(1) - y(0) = \underbrace{\int_0^1 1 + 2\sin(t^2)\,dt}$$

$y(1) - 3 = 1.621$

$y(1) = 4.621$

Slope $= 0.863$

point: $(9.314, 4.621)$

$\boxed{y - 4.621 = 0.863(x - 9.314)}$

(c) speed at $t=1$

$$\sqrt{(14\cos(1^2)\sin(e^1))^2 + (1+2\sin(1^2))^2} = \boxed{4.105}$$

(d) acceleration vector at $t=1$ = $\langle x''(1), y''(1) \rangle$

use math 8 on calc

$$\boxed{\langle -28.425, 2.161 \rangle}$$

(5) $x'(t) = \sqrt{3t}$ $\quad y'(t) = 3\cos\left(\frac{t^2}{2}\right)$

$x(4) = 1$, $y(4) = 5$

(a) acceleration vector at $t=4$ $\quad \langle x''(4), y''(4)\rangle$

math 8 on calc

$\boxed{\langle 0.433, -11.872\rangle}$

(b) $y(0) = ?$

$y(4) - y(0) = \underbrace{\int_0^4 3\cos\left(\frac{t^2}{2}\right) dt}$

$5 - y(0) = 3.399$

$-y(0) = -1.601$

$\boxed{y(0) = 1.601}$

(c) speed $= \underbrace{\sqrt{(\sqrt{3t})^2 + \left(3\cos\left(\frac{t^2}{2}\right)\right)^2}}_{y_1} = \underbrace{3.5}_{y_2}$

$\boxed{t = 2.226}$

(d) $\int_0^4 \sqrt{\left(\sqrt{3t}\right)^2 + \left(3\cos\left(\frac{t^2}{2}\right)\right)^2}\, dt = \boxed{13.182}$