AP Calculus BC
Unit 11 – Parametric Equations & Polar Coordinates

Day 2 Notes: Parametric Equations & Calculus

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DERIVATIVES OF PARAMETRIC EQUATIONS

If $x = f(t)$ and $y = g(t)$ represent curve C, then the slope of C at point (x, y) is given by

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \text{ where } \frac{dx}{dt} \neq 0.$$

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Example: Find the equation of the tangent to the curve defined by $x = \sqrt{t}$ and $y = \sqrt{t-1}$ when $t = 2$.

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HIGHER ORDER DERIVATIVES

1^{ST} derivative: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

2^{nd} derivative: $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[dy/dx\right]}{dx/dt}$

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Example: Find $\frac{d^2y}{dx^2}$ if $x = \theta - \sin\theta$ and $y = 1 - \cos\theta$.

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HORIZONTAL & VERTICAL TANGENTS

- **Horizontal:** Since $\frac{dy}{dx} = 0$ implies a horizontal tangent, and $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, horizontal tangents will occur when $\frac{dy}{dt} = 0$.
- **Vertical:** Vertical tangents occur where $\frac{dy}{dx}$ is undefined. This happens when $\frac{dx}{dt} = 0$.
- **Neither:** If $\frac{dy}{dt}$ and $\frac{dx}{dt}$ are simultaneously equal to 0 for the same value of *t*, we will usually have a sharp turn or a cusp, NOT a horizontal or vertical tangent.

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Example: Find all points of horizontal and vertical tangency to the curve given by $x = \cos\theta$, $y = 2\sin\theta$.

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Another example: The prolate cycloid given by $x = 2t - \pi \sin t$ and $y = 2 - \pi \cos t$ crosses over itself at the point (0, 2). (Graph to confirm! Set T-min and T-max so that $-2\pi \le t \le 2\pi$) Find the equations of both tangent lines at this point.

ARC LENGTH OF PARAMETRICALLY DEFINED CURVES

$$s = \int_{t_1}^{t_2} \sqrt{[x'(t)]^2 + [y'(t)]^2}\ dt$$

*Arc length gives the **total distance traveled**.

Example: Find the total distance a particle travels along a path given by $x = t^2 + 1$ and $y = 4t^3 + 3$ on the interval $-1 \le t \le 0$.

FINDING THE POSITION OF A PARTICLE

Position of the particle uses the ***Fundamental Theorem of Calculus!***

$$x(t_2) - x(t_1) = \int_{t_1}^{t_2} x'(t)\ dt$$

or

$$y(t_2) - y(t_1) = \int_{t_1}^{t_2} y'(t)\ dt$$

Example: The position of a particle in the *xy*-plane is given by $(x(t),\ y(t))$, with $\frac{dx}{dt} = t^2 + \sin(3t^2)$. At $t = 0$, the particle at the point (5, 1). Find the *x*-coordinate of the particle at $t = 3$.

SPEED OF A PARTICLE **Speed =** $\sqrt{[x'(t)]^2 + [y'(t)]^2}$	**Example:** A particle follows a path defined parametrically by $x(t) = 2\sqrt{t-3}$, $y(t) = 3t^2$. What is the speed of the particle at t = 7?
AREA OF PARAMETRICALLY DEFINED REGIONS $A = \int_a^b y\ dx$ *****Note:** Since we are integrating with respect to *x*, the limits ***a*** and ***b*** are *x*-values. Use these to find the corresponding values of $\boldsymbol{\theta}$ or ***t***.	**Example:** Find the area of the region enclosed by the graph of $x = \sin\theta$, $y = \sin^2\theta$, the *x*-axis and the vertical line *x* = 1.
VOLUME OF PARAMETRICALLY DEFINED REGIONS $V = \pi\int_a^b y^2\ dx$	**Example:** Suppose the region described in the previous example is rotated about the x-axis. Find the volume of the resulting solid.

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Unit 11 – Day 2 – Assignment

#'s 1 – 2: Find dy/dx and d^2y/dx^2. Then find the slope and concavity (if possible) at the Indicated value of the parameter.

1) $x = 2t, \quad y = 3t - 1, \quad t = 3$	**2)** $x = 2cos\theta, \quad y = 2sin\theta, \quad t = \pi/4$

#'s 3 & 4: Find all points (if any) of horizontal and vertical tangency to the curve.

3) $x = 1 - t, \; y = t^3 - 3t$	**4)** $x = 3cos\theta, \; y = 3sin\theta$

5) Find an equation of the tangent line at the point (0, 2) for $x = 2cot\theta$ and $y = 2sin^2\theta$.

6) Find the total distance a particle travels along a path by $x = t^2$ and $y = 2t$ on the interval $0 \le t \le 2$. (Calculator Active)

7) A particle follows a path defind parametrically by $x(t) = 4t^2 - 3$ and $y(t) = 2t^3$. What is the speed of the particle at $t = 3$? (Calculator Active)

8) Find the area of the region enclosed by the graph of $x = 2sin^2\theta$, $y = 2sin^2\theta tan\theta$, the x-axis and the vertical line $x = 2$. (Calculator Active)