AP Calculus BC **Name:** Answer Key*
Unit 11 – Day 2 – Assignment

#'s 1 – 2: Find dy/dx and d^2y/dx^2. Then find the slope and concavity (if possible) at the Indicated value of the parameter.

1) $x = 2t, \quad y = 3t - 1, \quad t = 3$

$\boxed{\frac{dy}{dx} = \frac{3}{2}} \rightarrow$ slope $= \frac{3}{2}$

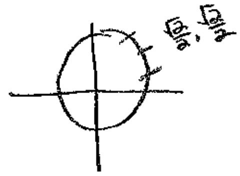
$\frac{d^2y}{dx^2} = \frac{0}{2} = \boxed{0}$

concavity $\rightarrow$ none

2) $x = 2\cos\theta, \quad y = 2\sin\theta, \quad t = \pi/4$

$\frac{dy}{dx} = \frac{2\cos\theta}{-2\sin\theta} = \boxed{-\cot\theta}$

slope $\rightarrow -\cot\left(\frac{\pi}{4}\right) = -1$



$\frac{d^2y}{dx^2} = \frac{\csc^2\theta}{-2\sin\theta} = \frac{\frac{1}{\sin^2\theta}}{-2\sin\theta} = \frac{1}{\sin^2\theta} \cdot \frac{1}{-2\sin\theta}$

$= \boxed{\frac{-1}{2\sin^3\theta}}$

$\frac{-1}{2\sin^3\left(\frac{\pi}{4}\right)} = \frac{-1}{2\sin^3\left(\frac{\sqrt{2}}{2}\right)} =$ negative

Concave down

#'s 3 & 4: Find all points (if any) of horizontal and vertical tangency to the curve.

3) $x = 1 - t, \; y = t^3 - 3t$

horizontal $\frac{dy}{dt} = 0$ $\quad 3t^2 - 3 = 0$

$3(t^2 - 1) = 0$

$3(t+1)(t-1) = 0$

$t = -1, \; t = 1$

$t = -1 \rightarrow \quad x = 1 - (-1) = 2$
$y = (-1)^3 - 3(-1) = 2 \quad \boxed{(2,2)}$

$t = 1 \rightarrow \quad x = 1 - 1 = 0$
$y = (1)^3 - 3(1) = -2 \quad \boxed{(0,-2)}$

vertical $\frac{dx}{dt} = 0$ $\quad -1 \neq 0$

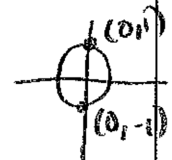
$\boxed{\text{none}}$

4) $x = 3\cos\theta, \; y = 3\sin\theta$

horiz: $\frac{dy}{dt} = 0$ $\quad 3\cos\theta = 0$

$\cos\theta = 0$

$\theta = \pi/2, \; 3\pi/2$



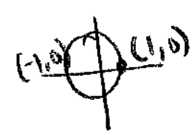
$\theta = \pi/2 \rightarrow x = 3\cos(\pi/2) = 0$
$y = 3\sin(\pi/2) = 3 \quad \boxed{(0,3)}$

$\theta = 3\pi/2 \rightarrow x = 3\cos(3\pi/2) = 0$
$y = 3\sin(3\pi/2) = -3 \quad \boxed{(0,-3)}$

vert. $\frac{dx}{dt} = 0$ $\quad -3\sin\theta = 0$

$\sin\theta = 0$

$\theta = 0, \pi$

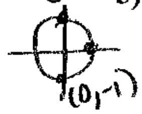


$\theta = 0 \rightarrow x = 3\cos(0) = 3$
$y = 3\sin(0) = 0 \quad \boxed{(3,0)}$

$\theta = \pi \rightarrow x = 3\cos\pi = -3 \quad \boxed{(-3,0)}$
$y = 3\sin\pi = 0$

$2(\sin\theta)^2$

(0,1) 5) **Find an equation of the tangent line at the point $\boxed{(0, 2)}$ for $x = 2\cot\theta$ and $y = 2\sin^2\theta$.**



(0,2)
solve for t.
$2 = 2\sin^2\theta$
$1 = \sin^2\theta$
$1 = \sin\theta$
$\theta = \pi/2$

$$\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{-2\csc^2\theta}$$

slope: $\dfrac{4\sin(\pi/2)\cos(\pi/2)}{-2\csc^2(\pi/2)} = \dfrac{4(1)(0)}{-2(1)^2} = 0$

$y - 2 = 0(x - 0)$
$y - 2 = 0$ $\boxed{y = 2}$

6) **Find the total distance a particle travels along a path by x = t² and y = 2t on the interval $0 \le t \le 2$. (Calculator Active)**

Total distance = Arc Length

$$\int_0^2 \sqrt{(2t)^2 + (2)^2}\, dt = \boxed{5.916}$$

7) **A particle follows a path defind parametrically by $x(t) = 4t^2 - 3$ and $y(t) = 2t^3$. What is the speed of the particle at $t = 3$? (Calculator Active)**

speed $= \sqrt{(8t)^2 + (6t^2)^2} = \sqrt{64t^2 + 36t^4}$

$t = 3 \rightarrow$ $\sqrt{64(3)^2 + 36(3)^4} = \boxed{59.093}$

$2(\sin\theta)^2$

8) **Find the area of the region enclosed by the graph of $x = 2\sin^2\theta$, $y = 2\sin^2\theta\tan\theta$, the x-axis and the vertical line x = 2. (Calculator Active)**

$X = 0$

$0 = 2\sin^2\theta$
$0 = \sin\theta$
$\theta = 0$

$2 = 2\sin^2\theta$
$1 = \sin\theta$
$\theta = \pi/2$

$$A = \int_0^2 y\, dx$$

$$A = \int_0^{\pi/2} 2\sin^2\theta\tan\theta\,(4\sin\theta)(\cos\theta)\,d\theta$$

$$\int_0^{\pi/2} 8\sin^3\theta\tan\theta\cos\theta\, d\theta$$

$$= \boxed{4.712}$$