AP Calculus BC
Unit 11 – Parametric Equations & Polar Coordinates

Day 1 Notes: Parametric Equations

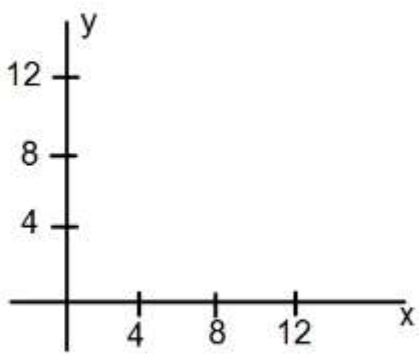
**Parametric Equations introduce a “parameter”, frequently *t* (time) or θ (angle).

** We can tell where an object will be and at what time it will be there.

**Parametric equations give position, speed, and direction.

__Example 1__: Use the table below to help you graph the parametric equations
$x(t) = t^2 + t$ and $y(t) = t^2 - t$.

t	-3	-2	-1	0	1	2	3
x							
y							



Note that when (x, y) is plotted according to asending values of *t*, the curve is traced out in a specific direction called the **__orientation__**.

__Example 2__: Graphing Parametric Equations in your calculator.
*Set your calculator to parametric mode.

a) Graph $x = t^2 - 4$, $y = t/2$, for $-2 \le t \le 3$

b) Graph $x = 4t^2 - 4$, $y = t$, for $-1 \le t \le 3/2$

There are times when we might want to convert parametric equations to rectangular, and vice versa.

1) **Solve one of the equations for t.**
2) **Substitute t into the other equation.**
3) **Adjust the domain of the rectangular equation to fit that of the parametric equations.**

Example 3: Convert x = t – 1 and y = t / (t – 1) to rectangular form.

Example 4: Convert x = 3t – 1 and y = 2t – 1 to rectangular form.

*****Remember: $cos^2\theta + sin^2\theta = 1$ and $sec^2\theta - tan^2\theta = 1$***

Example 5: Convert to rectangular form: x = 5cosθ, y = 5sinθ

Example 6: Convert to rectangular form: x = 4sin(2θ), y = 2cos(2θ)

Example 7: Convert to parametric form: $\frac{(x-4)^2}{25}+\frac{(y+1)^2}{9}=1$

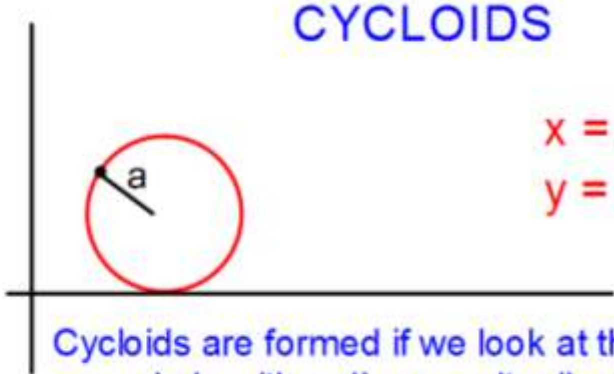
Example 8: Convert to parametric form: $\frac{y^2}{14}-\frac{x^2}{9}=1$

Definition of <u>SMOOTH CURVE</u>

If C is a curve represented by $x = f(t)$, $y = g(t)$ on an interval $[a, b]$, and f' and g' are continuous on $[a, b]$ and not simultaneously equal to 0 (except maybe at the endpoints of $[a, b]$), then C is a smooth curve.

*A curve is not smooth wherever it has cusps or sharp turns.

CYCLOIDS



$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

Cycloids are formed if we look at the motion of a point on a circle with radius a as it rolls along a line.

Cycloids have sharp turns at $x = 2n\pi a$.

Example 9: Sketch the cycloid $x = 4(\theta - \sin\theta)$, $y = 4(1 - \cos\theta)$. Identify any points at which the curve is not smooth.

AP Calculus BC **Name: ________________________________**
Unit 11 – Day 1 – Assignment

#'s 1 – 3: Sketch the curve (by hand) represented by the parametric equations and write the corresponding rectangular equation by eliminating the parameter.

1) $x = \sqrt{t}, y = t - 2$	2) $x = sec\theta, \quad y = cos\theta$
3) $x = 3cos\theta, \quad y = 3sin\theta$	

#'s 4 – 6: Use your graphing calculator to sketch the curve represented by the parametric equations. Eliminate the parameter and write the corresponding rectangular equation.

4) $x = 4 + 2cos\theta$ $y = -1 + sin\theta$	5) $x = t^3, y = 3lnt$

6)

$$x = e^{-t}, y = e^{3t}$$

#'s 7 – 8: Find a set of parametric equations for the conic.

7)	8)
$Ellipse: Vertices\ (\pm 5, 0), Foci\ (\pm 4, 0)$	$Hyperbola: Vertices\ (\pm 4, 0), Foci\ (\pm 5, 0)$

9) Graph the cycloid x = 2(θ – sinθ), y = 2(1 – cosθ). Identify any points at which the curve is not smooth.