Unit 10 - Review - Warm-up:

(a) $f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2!} + \frac{f'''(1)(x-1)^3}{3!}$

$1 + -\frac{1}{2}(x-1) + \frac{(-1)^2(2-1)!}{2^2} \cdot \frac{(x-1)^2}{2!} + \frac{(-1)^3(3-1)!}{2^3} \cdot \frac{(x-1)^3}{3!}$

$$\boxed{1 - \frac{1}{2}(x-1) + \frac{1}{2^2 \cdot 2!}(x-1)^2 - \frac{2!}{2^3 \cdot 3!}(x-1)^3}$$

0 1 2 3

$\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)! (x-1)^n}{2^n n!}$ or $\boxed{\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n n}}$

General term

$\frac{(n-\cancel{1})!}{n(n-\cancel{1})!}$

(b) radius = 2, center = 1

-1 1 3

$\underline{x=-1}$: $\sum_{n=1}^{\infty} \frac{(-1)^n(-2)^n}{2^n n} = \sum \frac{(-1)^n(-1)^n}{n} = \sum \frac{1}{n}$ p-series test
p=1
diverges

$\underline{x=3}$: $\sum_{n=1}^{\infty} \frac{(-1)^n \cancel{(2)^n}}{\cancel{2^n} n} = \sum \frac{(-1)^n}{n}$ alt. series test

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n+1} \le \frac{1}{n}$ ✓

Conv.

$\boxed{(-1, 3]}$

(c) $f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{2^2 \cdot 2!} \cdot (x-1)^2$

$f(1.2) \approx 1 - \frac{1}{2}(1.2-1) + \frac{1}{2^2 \cdot 2!}(1.2-1)^2 = \boxed{0.905}$

(d) Alternating series with terms that decrease to 0 if you take absolute value, so use the "next term".

$\left| \frac{2!}{2^3 \cdot 3!} (1.2-1)^3 \right| = \boxed{\frac{1}{3000}} \leq \frac{1}{1000}$ ✓