

Unit 10 - Review - Warm-up :

$$(a) f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2!} + \frac{f'''(1)(x-1)^3}{3!}$$

$$1 + -\frac{1}{2}(x-1) + \frac{(-1)^2(2-1)!}{2^2} \cdot \frac{(x-1)^2}{2!} + \frac{(-1)^3(3-1)!}{2^3} \cdot \frac{(x-1)^3}{3!}$$

$$\boxed{1 - \frac{1}{2}(x-1) + \frac{1!}{2^2 \cdot 2!} (x-1)^2 - \frac{2!}{2^3 \cdot 3!} (x-1)^3}$$

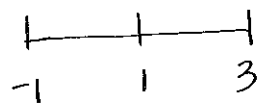
$$\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)! (x-1)^n}{2^n n!}$$

$$\text{or } \boxed{\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n n}}$$

General term

$$\frac{(n-1)!}{n(n-1)!}$$

(b) radius = 2, center = 1



$$\underline{x=-1}: \sum_{n=1}^{\infty} \frac{(-1)^n (-2)^n}{2^n n} = \sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n} \quad \begin{array}{l} \text{p-series test} \\ P=1 \\ \text{diverges} \end{array}$$

$$\underline{x=3}: \sum_{n=1}^{\infty} \frac{(-1)^n (2)^n}{2^n n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \quad \begin{array}{l} \text{alt. series test} \\ \lim_{n \rightarrow \infty} \frac{1}{n} = 0 \checkmark \end{array}$$

$$\frac{1}{n+1} \leq \frac{1}{n} \checkmark$$

Conv.

$$\boxed{(-1, 3]}$$

$$(c) \quad f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{2^2 \cdot 2!} \cdot (x-1)^2$$

$$f(1.2) \approx 1 - \frac{1}{2}(1.2-1) + \frac{1}{2^2 \cdot 2!} (1.2-1)^2 = \boxed{0.905}$$

(d) Alternating series with terms that decrease to 0 if you take absolute value, so use the "next term".

$$\left| \frac{2!}{2^3 \cdot 3!} (1.2-1)^3 \right| = \boxed{\frac{1}{3000}} \leq \frac{1}{1000} \quad \checkmark$$