Unit 10 - Review Scavenger Hunt

$f(x) = \sin 2x$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin 2x = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}$$

$\underline{n=0}$: $\frac{(-1)^0 (2x)^{2(0)+1}}{(2(0)+1)!} = \frac{(1)(2)x}{1} = 2x$

$\underline{n=1}$: $\frac{(-1)^1 (2x)^{2(1)+1}}{(2(1)+1)!} = \frac{(-1)(2^3)x^3}{3!} = \frac{-8x^3}{3!}$

$\underline{n=2}$: $\frac{(-1)^2 (2x)^{2(2)+1}}{(2(2)+1)!} = \frac{(1)(2^5)(x^5)}{5!} = \frac{32x^5}{5!}$

$$\boxed{2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \dots}$$

$$f(x) = e^{-2x}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-2x} = \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n (2)^n x^n}{n!}$$

$\underline{n=0}$: $$\frac{(-1)^0 (2)^0 x^0}{0!} = \frac{1}{1} = 1$$

$\underline{n=1}$: $$\frac{(-1)^1 (2)^1 x^1}{1!} = (-1)(2)x = -2x$$

$\underline{n=2}$: $$\frac{(-1)^2 (2)^2 x^2}{2!} = \frac{4x^2}{2!}$$

$\underline{n=3}$: $$\frac{(-1)^3 (2)^3 x^3}{3!} = \frac{-8x^3}{3!}$$

$\underline{n=4}$: $$\frac{(-1)^4 (2)^4 x^4}{4!} = \frac{16x^4}{4!}$$

$$\boxed{1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \frac{16x^4}{4!} - \ldots}$$

$f(x) = \dfrac{3}{3-x}$, centered at $x=1$

$$\frac{3}{3-(x-1)-1} = \frac{3}{2-(x-1)} = \frac{3/2}{1-\frac{(x-1)}{2}} \qquad a=\frac{3}{2} \quad r=\frac{x-1}{2}$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)\left(\frac{x-1}{2}\right)^n$$

$$\left|\frac{x-1}{2}\right| < 1$$

$$|x-1| < 2 \qquad R=2$$

-1 1 3

Geometric series

$$\boxed{(-1,3)}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad \frac{1}{(2n)!} x^{\cancel{2n}}$$

$$\cancel{(2n)}(2n-1)!$$

$$\boxed{f'(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}}$$

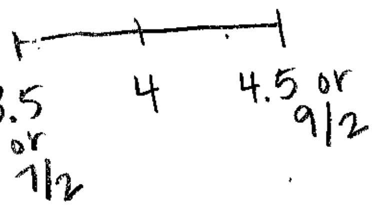
$$\sum_{n=0}^{\infty} \frac{2^n (x-4)^n}{n} \qquad \text{center} = 4$$

$$\lim_{n\to\infty} \left| \frac{\cancel{2^{n+1}}\,(x-4)^{\cancel{n+1}}}{(n+1)} \cdot \frac{n}{\cancel{2^n}\cancel{(x-4)^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{2(x-4)(n)}{(n+1)} \right| = |2(x-4)| \lim_{n\to\infty} \left| \frac{n}{n+1} \right| = |2(x-4)|$$

$$|2(x-4)| < 1$$

$$|x-4| < \frac{1}{2} \qquad R = 1/2$$



$\underline{x = 7/2}$: $$\sum_{n=0}^{\infty} \frac{2^n (-\frac{1}{2})^n}{n} = \sum \frac{(-1)^n}{n}$$ alt series test

$$\lim_{n\to\infty} \frac{1}{n} = 0 \checkmark$$

$$\frac{1}{n+1} \le \frac{1}{n} \checkmark$$

converges

$\underline{x = 9/2}$: $$\sum_{n=0}^{\infty} \frac{2^n (1/2)^n}{n} = \sum \frac{1}{n}$$ p-series test

diverges

$$\left[\frac{7}{2}, \frac{9}{2}\right)$$

$$\boxed{\frac{7}{2} \le x < \frac{9}{2}}$$

$$\sum_{n=0}^{\infty} \left(\frac{2x}{3}\right)^n$$

Geometric Series

$$\left|\frac{2x}{3}\right| < 1$$

$$|x| < \frac{3}{2}$$

$$\boxed{\text{Radius} = 3/2}$$

$$f(x) = e^{2x} + 3x \rightarrow e^{2(0)} + 3(0) = 1$$

$$f'(x) = 2e^{2x} + 3 \rightarrow 2e^{2(0)} + 3 = 5$$

$$f''(x) = 4e^{2x} \rightarrow 4e^{2(0)} = 4$$

$$f'''(x) = 8e^{2x} \rightarrow 8e^{2(0)} = 8$$

$$1 + 5x + \frac{4x^2}{2!} + \frac{8x^3}{3!} = \boxed{1 + 5x + 2x^2 + \frac{4}{3}x^3}$$

$$f(x) = e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots$$

$$\int_0^x e^{-t^2} dt = \int_0^x 1 - t^2 + \tfrac{1}{2}t^4 - \tfrac{1}{6}t^6 \, dt$$

$$t - \frac{t^3}{3} + \frac{t^5}{10} - \frac{t^7}{42} \Big]_0^x$$

$$\boxed{x - \tfrac{1}{3}x^3 + \tfrac{1}{10}x^5 - \tfrac{1}{42}x^7}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$\underline{n=0}$: $\frac{(-1)^0 x^{2(0)}}{(2 \cdot 0)!} = 1$

$\underline{n=1}$: $\frac{(-1)^1 x^{2(1)}}{(2 \cdot 1)!} = \frac{-x^2}{2!}$

$\underline{n=2}$: $\frac{(-1)^2 x^{2(2)}}{(2 \cdot 2)!} = \frac{x^4}{4!}$

$$f(x) = 1 - \tfrac{1}{2}x^2 + \tfrac{1}{24}x^4$$

$f'(x) = 0 - x + \frac{4}{24}x^3 \rightarrow f'(0) = 0$ critical point at $x=0$

$f''(x) = 0 - 1 + \frac{12}{24}x^2 \rightarrow f''(0) = -1$ negative

$\boxed{\text{relative maximum}}$

$$f(2) + \cancel{f'(2)(x-2)} + \frac{f''(2)(x-2)^2}{2!} + \cancel{\frac{f'''(x-2)^3}{3!}} + \frac{f^4(x-2)^4}{4!}$$

$$7 + \frac{(2-1)!}{3^2} \cdot \frac{(x-2)^2}{2!} + \frac{(4-1)!}{3^4} \frac{(x-2)^4}{4!}$$

$$7 + \frac{1}{3^2 \cdot 2!}(x-2)^2 + \frac{3!}{3^4 \cdot 4!}(x-2)^4$$