Unit 10 - Days 1-5 - Quiz Review

(1) $n=3$, Taylor Polynomial, $c=1$

$$f(x)=\frac{x+3}{x} \rightarrow f(1)=\frac{1+3}{1}=\boxed{4}$$

$$f'(x)=\frac{(x)(1)-(x+3)(1)}{x^2}=\frac{x-x-3}{x^2}=-3x^{-2} \rightarrow f'(1)=\frac{-3}{1^2}=\boxed{-3}$$

$$f''(x)=6x^{-3} \rightarrow f''(1)=\frac{6}{1^3}=\boxed{6}$$

$$f'''(x)=-18x^{-4} \rightarrow f'''(1)=\frac{-18}{1^4}=\boxed{-18}$$

$$P_3(x)=4-3(x-1)+\frac{6(x-1)^2}{2!}-\frac{18(x-1)^3}{\underset{3\cdot 2}{3!}}$$

$$\boxed{P_3(x)=4-3(x-1)+3(x-1)^2-3(x-1)^3}$$

(2) Maclaurin Polynomial $\rightarrow c=0$, $n=3$

$$f(x)=\sin x \rightarrow f(0)=\sin 0=\boxed{0}$$
$$f'(x)=\cos x \rightarrow f'(0)=\cos 0=\boxed{-1}$$
$$f''(x)=-\sin x \rightarrow f''(0)=-\sin 0=\boxed{0}$$
$$f'''(x)=-\cos x \rightarrow f'''(0)=-\cos 0=\boxed{-1}$$

$$P_3(x)=\cancel{0}+-1x+\cancel{\frac{0x^2}{2!}}-1\frac{x^3}{3!}$$

$$P_3(x)=-x-\frac{x^3}{3!} \qquad \boxed{\frac{-x^3}{3!}}$$

↑ 2nd term

(3) $e^x = f(x)$ 3rd degree Maclaurin

$f(x) = e^x \rightarrow f(0) = e^0 = \boxed{1}$
$f'(x) = e^x \rightarrow f'(0) = e^0 = \boxed{1}$
$f''(x) = e^x \rightarrow f''(0) = e^0 = \boxed{1}$
$f'''(x) = e^x \rightarrow f'''(0) = e^0 = \boxed{1}$

$$P_3(x) = 1 + 1x + \frac{1x^2}{2!} + \frac{1x^3}{3!}$$ 3·2·1

$$P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

$$P_3(0.2) = 1 + 0.2 + \frac{1}{2}(0.2)^2 + \frac{1}{6}(0.2)^3$$
$$= \boxed{1.221}$$

use your calc.

$$R_3(x) = \frac{f^4(x)}{4!}(x-0)^4$$ max

$$= \frac{e^{0.2}}{4!}(0.2)^4$$

$$= \boxed{8.143 \times 10^{-5}}$$

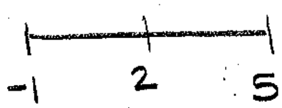
4) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ $\boxed{\text{center} = 2}$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{\cancel{n+1}}}{(n+1)3^{\cancel{n+1}}} \cdot \frac{n\cancel{3^n}}{\cancel{(x-2)^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(x-2)(n)}{3(n+1)} \right| = \left| \frac{x-2}{3} \right| \lim_{n\to\infty} \left| \frac{n}{n+1} \right| = \left| \frac{x-2}{3} \right|$$

$$\left| \frac{x-2}{3} \right| < 1$$

$$|x-2| < 3 \quad \boxed{R=3}$$



$\underline{x=-1}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1-2)^n}{n3^n} = \sum \frac{(-3)^n}{n\,3^n} = \sum \frac{(-1)^n}{n}$

Alt. Series Test

$\displaystyle\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\displaystyle\frac{1}{n+1} \le \frac{1}{n}$ ✓ converges

$\underline{x=5}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(5-2)^n}{n3^n} = \sum \frac{\cancel{3^n}}{n\cancel{3^n}} = \sum \frac{1}{n}$

p-series test

p=1 diverges

$\boxed{[-1, 5)}$

⑤ $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\frac{1}{(2n+1)!} x^{2n+1}$

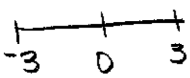
$= \frac{1}{(2n+1)(2n!)} x^{2n+1}$

$$\boxed{f'(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n!)}}$$

⑥ $f(x) = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^n$ center=0

$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}} \left(\frac{x}{3}\right)^{n+1}}{\cancel{(-1)^n} \cancel{\left(\frac{x}{3}\right)^n}} \right| = \lim_{n\to\infty} \left|\frac{x}{3}\right| = \left|\frac{x}{3}\right|$

$\left|\frac{x}{3}\right| < 1$ $|x| < 3$ $R=3$



$\int f(x)dx = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{3}\right)^{n+1} (3)}{(n+1)}$

$\underline{x=-3}$: $\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{-3}{3}\right)^{n+1} (3)}{(n+1)} = \sum \frac{(-1)^n (-1)^{n+1} (3)}{n+1}$

$= \sum \frac{(-1)^n (-1)^n (-1)^1 (3)}{n+1} = \sum \frac{-3}{n+1}$ $a_n = \frac{-3}{n+1}$ $b_n = \frac{-3}{n}$

bigger

$\sum \frac{-3}{n}$ p-series test

p=1 diverges

∴ diverges by Direct Comparison Test

$\underline{X=3}$: $\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{3}{3}\right)^{n+1}(3)}{(n+1)} = \sum \frac{(-1)^n (1)^{n+1}(3)}{(n+1)}$

$= \sum \frac{(-1)^n (1)^n (1)^1 (3)}{(n+1)} = \sum \frac{(-1)^n (3)}{(n+1)}$

Alt Series Test

$\lim_{n\to\infty} \frac{3}{n+1} = 0$ ✓

$\frac{3}{n+2} \le \frac{3}{n+1}$ ✓ Converges

$\boxed{(-3, 3]}$

⑦ $f(x) = \frac{2}{4x+3}$ $C=1$

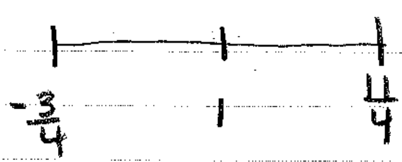
$\frac{2}{4x+3} = \frac{2}{3+4x} \longrightarrow \frac{2}{3+4(x-1)+4} = \frac{2}{7+4(x-1)} = \frac{2/7}{1 + \frac{4(x-1)}{7}}$

$a = 2/7$

$r = \frac{-4(x-1)}{7}$

$\boxed{\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)\left(\frac{-4(x-1)}{7}\right)^n}$

$\left|\frac{-4(x-1)}{7}\right| < 1$



$|x-1| < \frac{7}{4}$ $\boxed{R = \frac{7}{4}}$

$\frac{4}{4} + \frac{7}{4}$

$\frac{4}{4} - \frac{7}{4}$

$\boxed{\left(-\frac{3}{4}, \frac{11}{4}\right)}$