Unit 10 – Day 6 – Assignment **Name:** Answer Key *

Use the definition to find the Taylor series (centered at c) for the function.

1) $f(x)=e^{2x}, c=0$ $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$ or	2) $f(x)=\cos x, c=\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}\sum_{n=0}^{\infty} \frac{(-1)^{n(n+1)/2}(x-\frac{\pi}{4})^n}{n!}$
3) $f(x)=\ln x, c=1$ $\sum_{n=0}^{\infty} \frac{(-1)^n(x-1)^{n+1}}{n+1}$	4) $f(x)=\sin 2x, c=0$ $\sum_{n=0}^{\infty} \frac{(-1)^n(2x)^{2n+1}}{(2n+1)!}$

Find the Maclaurin series for x the function. (Use the table of power series.)

5) $f(x) = e^{x^2/2}$

$$\sum_{n=0}^{\infty} \frac{(x^{2n})}{2^n n!}$$

6) $f(x) = \cos(x^{3/2})$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x^{3n})}{(2n)!}$$

7) $f(x) = \sin 2x$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}$$

8) $f(x) = \cos^2 x$

(Hint: $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$)

$$\frac{1}{2}\left[1 + \sum_{n=0}^{\infty} \frac{(-1)(2x)^{2n}}{(2n)!}\right]$$

Unit 10 - Day 6 Assignment

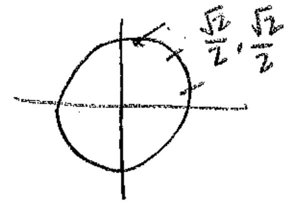
① $f(x) = e^{2x}$, $c = 0$

$f(x) = e^{2x} \rightarrow e^0 = 1$
$f'(x) = 2e^{2x}$ 2
$f''(x) = 4e^{2x}$ 4
$f'''(x) = 8e^{2x}$ 8

$$\boxed{\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}}$$

⓪ ① ② ③

$$1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!}$$



② $f(x) = \cos x$, $c = \pi/4$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n!)}$$

$f(x) = \cos x \rightarrow \cos(\pi/4) = \frac{\sqrt{2}}{2}$
$f'(x) = -\sin x \rightarrow -\sin(\pi/4) = -\frac{\sqrt{2}}{2}$
$f''(x) = -\cos x$ $-\frac{\sqrt{2}}{2}$
$f'''(x) = \sin x$ $\frac{\sqrt{2}}{2}$
$f^4(x) = -\cos x$ $-\frac{\sqrt{2}}{2}$

⓪ ① ② ③ ④

$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}(x-\tfrac{\pi}{4}) - \frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^2}{2!} + \frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^3}{3!} - \frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^4}{4!} + \ldots$$

$$\boxed{\frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} \frac{(x-\pi/4)^n (-1)^{n(n+1)/2}}{n!}}$$

note: $(-1)^{n(n+1)/2} = 1, -1, -1, 1, 1, -1, -1, \ldots$

(3) $f(x) = \ln x, \quad c = 1$

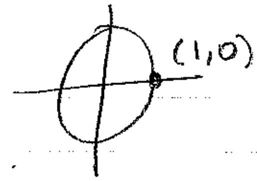
$$f(x) = \ln x \rightarrow \ln(1) = 0$$
$$f'(x) = 1/x = x^{-1} \rightarrow 1/1 = 1$$
$$f''(x) = -1x^{-2} = -1/x^2 \rightarrow -1/1^2 = -1$$
$$f'''(x) = 2x^{-3} = 2/x^{-3} \rightarrow 2/1^3 = 2$$
$$f^4(x) = -6x^{-4} \quad -6/x^4 \rightarrow -6$$

$$\cancel{0} + \overset{0}{1(x-1)} - \overset{1}{\frac{1(x-1)^2}{2!}} + \overset{2}{\frac{2(x-1)^3}{3!}} - \overset{3}{\frac{6(x-1)^4}{4!}} + \dots$$

$$\overset{0}{(x-1)} + \overset{1}{\frac{(x-1)^2}{2}} + \overset{2}{\frac{(x-1)^3}{3}} - \overset{3}{\frac{(x-1)^4}{4}}$$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n+1}}$$

4. $f(x) = \sin 2x, \quad C = 0$



$$f(x) = \sin 2x = 0$$
$$f'(x) = 2\cos 2x = 2$$
$$f''(x) = -4\sin 2x = 0$$
$$f'''(x) = -8\cos 2x = -8$$
$$f^4(x) = 16\sin 2x = 0$$
$$f^5(x) = 32\cos 2x = 32$$
$$f^6(x) = -64\sin 2x = 0$$
$$f^7(x) = -128\cos 2x = -128$$

$$\cancel{0} + 2x + \cancel{\frac{0x^2}{2!}} - \frac{8x^3}{3!} + \cancel{\frac{0x^4}{4!}} + \frac{32x^5}{5!} + \cancel{\frac{0x^6}{6!}} - \frac{128x^7}{7!} + \ldots$$

0 1 2 3

$$2x - \frac{8}{3!}x^3 + \frac{32}{5!}x^5 + \frac{-128}{7!}x^7$$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}}$$

⑤ $f(x) = e^{x^2/2}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} \qquad \left(\frac{x^2}{2}\right)^n$$

$$e^{x^2/2} = \sum_{n=0}^{\infty} \frac{(x^2/2)^n}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{(x^{2n})}{2^n n!}}$$

⑥ $f(x) = \cos x^{3/2}$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos x^{3/2} \sum_{n=0}^{\infty} \frac{(-1)^n (x^{3/2})^{2n}}{(2n!)} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n (x^{3n})}{(2n!}\Big)}$$

⑦ $f(x) = \sin 2x$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin 2x = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}}$$

⑧ $f(x) = \cos^2 x$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x) \qquad = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos 2x = \sum_{n=0}^{\infty} \frac{(-1)(2x)^{2n}}{(2n)!}$$

$$\boxed{\frac{1}{2}\left[1 + \sum_{n=0}^{\infty} \frac{(-1)(2x)^{2n}}{(2n)!}\right]}$$