

Day 5 Notes: Geometric Power Series

We can write a power series for some functions in the form of a geometric series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \text{ where } |r| < 1. \text{ We may have to manipulate } f(x) \text{ to put it in the form } \frac{a}{1-r}.$$

$$S = \frac{a}{1-r}$$

Examples:

1. Write a geometric power series centered at $c=0$ for $f(x) = \frac{3}{2x-1}$.

$$\frac{3}{2x-1} = \frac{3}{-1+2x} = \frac{-3}{1-2x} \quad a = -3 \quad r = (2x)$$

$$\sum_{n=0}^{\infty} (-3)(2x)^n$$

2. Find a geometric power series centered at $c = -2$ for $f(x) = \frac{3}{4-x}$.

****Take care of the center first!**

$$\frac{3}{4-x} \rightarrow \frac{3}{4-(x+2)+2} = \frac{3}{6-(x+2)} = \frac{3/6}{\frac{6}{6} - \frac{(x+2)}{6}} = \frac{1/2}{1 - \frac{(x+2)}{6}} \quad a = \frac{1}{2} \quad r = \frac{(x+2)}{6}$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{x+2}{6}\right)^n$$

3. Write a geometric power series centered at $c=0$ for $f(x) = \frac{4x-7}{2x^2+3x-2}$. Partial Fractions

$$\frac{4x-7}{(x+2)(2x-1)} = \frac{A}{x+2} + \frac{B}{2x-1}$$

$$4x-7 = A(2x-1) + B(x+2)$$

$$x = \frac{1}{2}: 4\left(\frac{1}{2}\right) - 7 = B\left(\frac{1}{2} + 2\right)$$

$$B = -2$$

$$x = -2: 4(-2) - 7 = A(2(-2) - 1)$$

$$A = 3$$

$$\frac{3}{x+2} + \frac{-2}{2x-1}$$

$$\frac{3}{x+2} = \frac{3}{2+x} = \frac{3/2}{1+x/2} \quad a = 3/2 \quad r = -x/2$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right) \left(-\frac{x}{2}\right)^n$$

$$\frac{-2}{2x-1} = \frac{-2}{-1+2x} = \frac{2}{1-2x} \quad a = 2 \quad r = (2x)$$

$$\sum_{n=0}^{\infty} (2)(2x)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right) \left(-\frac{x}{2}\right)^n + \sum_{n=0}^{\infty} (2)(2x)^n = \sum_{n=0}^{\infty} \left[\left(\frac{3}{2}\right) \left(-\frac{1}{2}\right)^n + (2)(2^n) \right] x^n$$

Convergence of Geometric Power Series

Let $\sum_{n=0}^{\infty} ar^n$ be a geometric power series.

- A geometric series will converge when $|r| < 1$.
- If a geometric series is centered at $x = c$ and the radius of convergence is R , the interval of convergence is $(c - R, c + R)$.
- A geometric series will never converge at the endpoints of the interval of convergence.

(,)

Examples:

4. Find a power series for $f(x) = \frac{4}{3x+2}$ centered at $c=2$. Then find the interval of convergence.

$$\frac{4}{3x+2} \rightarrow \frac{4}{2+3x} \rightarrow \frac{4}{2+3(x-2)+6} = \frac{4}{8+3(x-2)} = \frac{4/8}{8/8+3(x-2)/8} = \frac{1/2}{1+\frac{3(x-2)}{8}}$$

$$a = 1/2$$

$$r = \frac{-3(x-2)}{8}$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{-3(x-2)}{8}\right)^n$$

$$\left| \frac{-3(x-2)}{8} \right| < 1$$

$$|x-2| < \frac{8}{3}$$

$$R = \frac{8}{3}$$

$$\text{interval: } \left(2 - \frac{8}{3}, 2 + \frac{8}{3}\right)$$

5. Write a geometric power series centered at $c=0$ for $f(x) = \frac{3x-1}{x^2-1}$. Then find the interval of convergence.

$$\frac{3x-1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$3x-1 = A(x-1) + B(x+1)$$

$$x=1 \quad 3(1)-1 = B(1+1)$$

$$B = 1$$

$$x=-1 \quad 3(-1)-1 = A(-1-1)$$

$$A = 2$$

$$\frac{2}{x+1} + \frac{1}{x-1}$$

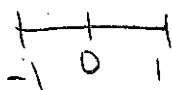
$$\frac{2}{x+1} = \frac{2}{1+x} \quad a=2 \quad r=-x$$

$$\sum_{n=0}^{\infty} (2)(-x)^n$$

$$\frac{1}{x-1} = \frac{1}{-1+x} = \frac{-1}{1-x} \quad a=-1 \quad r=x$$

$$\sum_{n=0}^{\infty} (-1)(x)^n$$

$$\sum_{n=0}^{\infty} (2)(-x)^n + \sum_{n=0}^{\infty} (-1)(x)^n = \sum_{n=0}^{\infty} [(2)(-1)^n - 1] x^n$$



$$(-1, 1)$$

$$|x| < 1 \quad R=1$$