

AP Calculus BC

Unit 10 – Sequences & Series (Part 2)

Day 5 Notes: Geometric Power Series

We can write a power series for some functions in the form of a geometric series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \text{ where } |r| < 1. \text{ We may have to manipulate } f(x) \text{ to put it in the form } \frac{a}{1-r}.$$

Examples:

1. Write a geometric power series centered at $c = 0$ for $f(x) = \frac{3}{2x-1}$.

2. Find a geometric power series centered at $c = -2$ for $f(x) = \frac{3}{4-x}$.

****Take care of the center first!**

3. Write a geometric power series centered at $c = 0$ for $f(x) = \frac{4x-7}{2x^2+3x-2}$.

Convergence of Geometric Power Series

Let $\sum_{n=0}^{\infty} ar^n$ be a geometric power series.

- A geometric series will converge when $|r| < 1$.
- If a geometric series is centered at $x = c$ and the radius of convergence is R , the interval of convergence is $(c - R, c + R)$.
- A geometric series will never converge at the endpoints of the interval of convergence.

Examples:

4. Find a power series for $f(x) = \frac{4}{3x+2}$ centered at $c = 2$. Then find the interval of convergence.

5. Write a geometric power series centered at $c = 0$ for $f(x) = \frac{3x-1}{x^2-1}$. Then find the interval of convergence.

AP Calculus BC
Unit 10 – Day 5 – Assignment

Name: _____

Find a power series for the function, centered at c , and determine the interval of convergence.

1) $f(x) = \frac{4}{5-x}, c = -2$

2) $f(x) = \frac{3}{2x-1}, c = 2$

3) $f(x) = \frac{1}{2x-5}, c = 0$

4) $f(x) = \frac{4}{3x+2}, c = 2$

5) $f(x) = \frac{3x}{x^2 + x - 2}, c = 0$

6) $f(x) = \frac{2}{1 - x^2}, c = 0$