AP Calculus BC
Unit 10 – Day 5 – Assignment

Name: Answer Key *

Find a power series for the function, centered at c, and determine the interval of convergence.

1) $f(x)=\frac{4}{5-x}, c=-2$ $\sum_{n=0}^{\infty}\left(\frac{4}{7}\right)\left(\frac{x+2}{7}\right)^n$ $R=7$ $(-9,5)$	2) $f(x)=\frac{3}{2x-1}, c=2$ $\sum_{n=0}^{\infty}(1)\left(-\frac{2}{3}(x-2)\right)^n$ $R=\frac{3}{2}$ $\left(\frac{1}{2},\frac{7}{2}\right)$
3) $f(x)=\frac{1}{2x-5}, c=0$ $\sum_{n=0}^{\infty}\left(-\frac{1}{5}\right)\left(\frac{2}{5}x\right)^n$ $R=5/2$ $\left(-\frac{5}{2},\frac{5}{2}\right)$	4) $f(x)=\frac{4}{3x+2}, c=2$ $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(-\frac{3}{8}(x-2)\right)^n$ $R=\frac{8}{3}$ $\left(-\frac{2}{3},\frac{14}{3}\right)$

5) $f(x)=\dfrac{3x}{x^2+x-2}$, $c=0$	**6)** $f(x)=\dfrac{2}{1-x^2}$, $c=0$
$\sum_{n=0}^{\infty}\left[\left(-\frac{1}{2}\right)^n-1\right]x^n$	$\sum_{n=0}^{\infty}\left[(-1)^n+1\right]x^n$
$R=1$	$R=1$
$(-1,1)$	$(-1,1)$

① $f(x) = \frac{4}{5-x}$, $c = -2$

$$\frac{4}{5-(x+2)+2} = \frac{4}{7-(x+2)} = \frac{4/7}{\frac{7}{7} - \frac{(x+2)}{7}}$$

$$= \frac{4/7}{1 - \frac{(x+2)}{7}} \qquad a = 4/7 \quad r = \frac{(x+2)}{7}$$

$$\boxed{\sum_{n=0}^{\infty} \left(\frac{4}{7}\right)\left(\frac{x+2}{7}\right)^n}$$

$$\left|\frac{x+2}{7}\right| < 1$$

$$|x+2| < 7 \qquad R = 7$$

$-9 \quad -2 \quad 5$

$$\boxed{(-9, 5)}$$

② $f(x) = \frac{3}{2x-1}$ $\quad c = 2$

$$\frac{3}{-1+2x} = \frac{3}{-1+2(x-2)+4} = \frac{3/3}{\frac{3+2(x-2)}{3}}$$

$2 + \frac{3}{2}$

$\frac{4}{2} + \frac{3}{2}$

$$= \frac{1}{1 + \frac{2(x-2)}{3}} \qquad a = 1 \quad r = -\frac{2(x-2)}{3}$$

$$\boxed{\sum_{n=0}^{\infty} (1)\left(-\frac{2}{3}(x-2)\right)^n}$$

$$\left|-\frac{2}{3}(x-2)\right| < 1 \qquad |x-2| < \frac{3}{2} \qquad R = \frac{3}{2}$$

$1/2 \quad 2 \quad 7/2$

$$\boxed{\left(\frac{1}{2}, 7/2\right)}$$

(3) $h(x) = \dfrac{1}{2x-5}$, $c=0$

$$\frac{1}{-5+2x} = \frac{-1/5}{1 + \frac{2x}{-5}} \qquad a = -1/5 \qquad r = \frac{2}{5}x$$

$$\boxed{\sum_{n=0}^{\infty} \left(-\frac{1}{5}\right)\left(\frac{2}{5}x\right)^n}$$

$$\left|\frac{2}{5}x\right| < 1$$

$$|x| < \frac{5}{2} \qquad R = 5/2$$

$-5/2 \qquad 0 \qquad 5/2$

$$\boxed{(-5/2, 5/2)}$$

(4) $f(x) = \dfrac{4}{3x+2}$, $c = 2$

$$\frac{4}{2+3x} \to \frac{4}{2+3(x-2)+6} \to \frac{4}{8+3(x-2)}$$

$$\to \frac{4/8}{1 + \frac{3(x-2)}{8}} \qquad a = 1/2 \qquad r = -\frac{3}{8}(x-2)$$

$$\boxed{\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(-\frac{3}{8}(x-2)\right)^n}$$

$$\left|-\frac{3}{8}(x-2)\right| < 1$$

$$|x-2| < \frac{8}{3} \qquad R = \frac{8}{3}$$

$\frac{6}{3} + \frac{8}{3}$

$\frac{6}{3} - \frac{8}{3}$

$-2/3 \qquad 2 \qquad 14/3$

$$\boxed{\left(-\frac{2}{3}, \frac{14}{3}\right)}$$

(5) $g(x) = \dfrac{3x}{x^2+x-2}$, $c=0$

$$\frac{3x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$3x = A(x-1) + B(x+2)$$

$\underline{x=1}$ $\quad 3(1) = B(1+2)$

$$3 = 3B$$

$$B = 1$$

$\underline{x=-2:}$ $\quad 3(-2) = A(-2-1)$

$$-6 = -3A$$

$$A = 2$$

$$\frac{2}{x+2} + \frac{1}{x-1}$$

$$\frac{2}{x+2} = \frac{2}{2+x} = \frac{2/2}{\frac{2+x}{2}} = \frac{1}{1+\frac{x}{2}} \qquad \begin{array}{l} a=1 \\ r=-x/2 \end{array}$$

$$\sum_{n=0}^{\infty} (1)\left(-\frac{x}{2}\right)^n$$

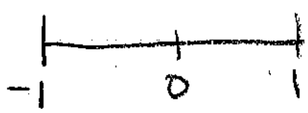
$$\frac{1}{x-1} = \frac{1}{-1+x} = \frac{-1}{1-x} \qquad \begin{array}{l} a=-1 \\ r=x \end{array}$$

$$\sum_{n=0}^{\infty} (-1)(x)^n$$

$$\sum_{n=0}^{\infty}\left(\frac{-x}{2}\right)^n + \sum_{n=0}^{\infty}(-1)(x^n) = \sum_{n=0}^{\infty}\left[\left(\frac{-x}{2}\right)^n - 1(x^n)\right] \longrightarrow$$

$$\sum_{n=0}^{\infty} \left[\left(-\frac{x}{2}\right)^n - 1(x^n)\right]$$

$$= \boxed{\sum_{n=0}^{\infty} \left[\left(-\frac{1}{2}\right)^n - 1\right] x^n} \qquad |x| < 1 \quad R = 1$$



$\boxed{(-1, 1)}$

⑥ $f(x) = \dfrac{2}{1-x^2}, \quad c = 0$

$$\frac{2}{(1+x)(1-x)} = \frac{A}{1+x} + \frac{B}{1-x} \rightarrow \frac{1}{1+x} + \frac{1}{1-x}$$

$2 = A(1-x) + B(1+x)$

$\underline{x=1}$ $\quad 2 = B(1+1)$

$B = 1$

$\underline{x=-1}$ $\quad 2 = A(1--1)$

$A = 1$

$$\frac{1}{1+x} \quad a = 1 \quad r = -x \qquad \sum_{n=0}^{\infty} (1)(-x)^n$$

$$\frac{1}{1-x} \quad a = 1 \quad r = x \qquad \sum_{n=0}^{\infty} (1)(x)^n$$

$$\sum_{n=0}^{\infty} (-x)^n + \sum_{n=0}^{\infty} x^n = \boxed{\sum_{n=0}^{\infty} \left[(-1)^n + 1\right] x^n}$$

$|x| < 1 \quad R = 1 \qquad \boxed{(-1, 1)}$