AP Calculus BC
Unit 10 – Sequences & Series (Part 2)

Day 4 Notes: Power Series (Part 2)

Properties of Functions Defined by Power Series

If $f(x) = a_n(x-c)^n$ has a radius of convergence R > 0, then on the interval ($c - R$, $c + R$), f is continuous and differentiable.

1. $f'(x) = \sum_{n=1}^{\infty} na_n(x-c)^{n-1}$

2. $\int f(x)\,dx = C + \sum a_n \cdot \frac{(x-c)^{n+1}}{n+1}$

***The radius of convergence is the same for $f(x)$, $f'(x)$, and $\int f(x)\,dx$. However, the interval of convergence may differ at the endpoints. **Always check for converge at the endpoints of the interval!**

Example 1: Given $f(x)$, find $f'(x)$ and $\int f(x)\,dx$

a) $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n}$

c) $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$

<u>Example 2:</u> Find the intervals of convergence for $f(x)$, $f'(x)$, and $\int f(x)\,dx$ if

$$f(x)=\sum_{n=1}^{\infty}\frac{(2x)^n}{n}$$

Example 3: Find the intervals of convergence for $f(x)$, $f'(x)$, and $\int f(x)\,dx$ if

$$f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n}$$

Unit 10 – Day 4 Assignment:

Find the intervals of convergence for $f(x)$, $f'(x)$, and $\int f(x)\,dx$ if…

1) $f(x)=\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$

2) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n5^n}$

3) $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$