Unit 10 - Day 4 Assignment

1 (a) $\boxed{f(x)} = \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$ $\quad C=0$

$$\lim_{n\to\infty} \left| \frac{\left(\frac{x}{2}\right)^{n+1}}{\left(\frac{x}{2}\right)^n} \right| = \lim_{n\to\infty} \left|\frac{x}{2}\right| = \left|\frac{x}{2}\right| \lim_{n\to\infty} 1 = \left|\frac{x}{2}\right|$$

$\left|\frac{x}{2}\right| < 1$

$|x| < 2$ $\quad R=2$

$-2 \qquad 0 \qquad 2$

$\underline{x=-2}$: $\sum_{n=0}^{\infty} \left(\frac{-2}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n$ Alt. Series Test $\lim_{n\to\infty} (1) = 1 \neq 0$ divg. by nth term

$\underline{x=2}$: $\sum_{n=0}^{\infty} \left(\frac{2}{2}\right)^n = \sum_{n=0}^{\infty} (1)^n$ geometric $\rightarrow$ diverges

$\boxed{f(x)} \rightarrow \boxed{(-2,2)}$

(b) $\boxed{f'(x):}$ $\sum_{n=0}^{\infty} n\left(\frac{x}{2}\right)^{n-1}\left(\frac{1}{2}\right) = \sum_{n=0}^{\infty} \left(\frac{n}{2}\right)\left(\frac{x}{2}\right)^{n-1}$

$\underline{x=-2}$: $\sum_{n=0}^{\infty} \left(\frac{n}{2}\right)\left(\frac{-2}{2}\right)^{n-1} = \sum_{n=0}^{\infty} \left(\frac{n}{2}\right)(-1)^{n-1}$ alt. series $\lim_{n\to\infty} \frac{n}{2} = \infty$

diverges by nth term

$\underline{x=2}$: $\sum_{n=0}^{\infty} \left(\frac{n}{2}\right)\left(\frac{2}{2}\right)^{n-1} = \sum_{n=0}^{\infty} \left(\frac{n}{2}\right)(1)^{n-1} = \sum_{n=0}^{\infty} \left(\frac{n}{2}\right)$

$\lim_{n\to\infty} \left(\frac{n}{2}\right) = \infty$ diverges by nth term

$\boxed{(-2,2)}$

$\left(\frac{x}{2}\right)^n$

ⓒ $\boxed{\int f(x)\,dx}$ $\quad \frac{(2)\left(\frac{x}{2}\right)^{n+1}}{n+1} \quad \sum_{n=0}^{\infty}\left(\frac{2}{n+1}\right)\left(\frac{x}{2}\right)^{n+1}$

$\underline{x=-2}$: $\sum_{n=0}^{\infty}\left(\frac{2}{n+1}\right)\left(\frac{-2}{2}\right)^{n+1} = \sum_{n=0}^{\infty}\left(\frac{2}{n+1}\right)(-1)^{n+1}$

Alt. Series

$\lim_{n\to\infty} \frac{2}{n+1} = 0$ ✓

$\frac{2}{n+2} \le \frac{2}{n+1}$ ✓ Conv.

$\underline{x=2}$: $\sum_{n=0}^{\infty}\left(\frac{2}{n+1}\right)\left(\frac{2}{2}\right)^{n+1} = \sum_{n=0}^{\infty}\left(\frac{2}{n+1}\right)$ $\quad a_n = \frac{2}{n+1} \quad b_n = \frac{2}{n}$

Limit Comp. Test

$\lim_{n\to\infty}\left|\frac{2}{n}\cdot\frac{n+1}{2}\right| = \lim_{n\to\infty}\frac{n+1}{n} = 1$ ✓

$\boxed{[-2,2)}$ $\quad \sum_{n=0}^{\infty}\frac{2}{n}$ diverges by p-series test

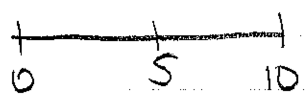
② ⓐ $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$ $C=5$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+2}}(x-5)^{\cancel{n+1}}}{(n+1)5^{\cancel{n+1}}} \cdot \frac{n\cancel{5^n}}{\cancel{(-1)^{n+1}}\cancel{(x-5)^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(x-5)(n)}{(n+1)5} \right| = \left| \frac{x-5}{5} \right| \lim_{n\to\infty} \left| \frac{n}{n+1} \right|$$

$$\left| \frac{x-5}{5} \right| \cdot 1 = \left| \frac{x-5}{5} \right|$$

$$\left| \frac{x-5}{5} \right| < 1$$



$|x-5| < 5$ $\boxed{R=5}$

$\underline{x=0}$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5)^n}{n5^n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^n}{n} \to \sum_{n=1}^{\infty} \frac{-1}{n}$

$(-1)^3(-1)^2 = (-1)(1) = -1$
$(-1)^4(-1)^3 = (+1)(-1) = -1$

p-series test
diverges

$\underline{x=10}$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cancel{(5)^n}}{n\cancel{5^n}}$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ Alt. Series Test

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n+1} \le \frac{1}{n}$ ✓
converges

$\boxed{f(x)} \to \boxed{[0, 10]}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n} \qquad \frac{1}{n5^n}(x-5)^n$$

ⓑ $$f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\,(x-5)^{n-1}}{5^n}$$

$\underline{x=0}$: $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5)^{n-1}}{5^n} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5)^n(-5)^{-1}}{(5^n)}$$

$-5^n \cdot -5^{-1}$ $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^n}{-5} = \sum_{n=1}^{\infty} \frac{-1}{-5} \qquad \lim_{n\to\infty} \frac{1}{5} \neq 0$$ diverg by nth term

$\underline{x=10}$: $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cancel{(5)}^{n-1}}{\cancel{5^n}} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{5}$$ Alt Series Test

$$\lim_{n\to\infty} \frac{1}{5} = \frac{1}{5} \neq 0$$

Diverges by nth term

$\boxed{(0, 10)}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n} \qquad \frac{1}{n5^n}(x-5)^n$$

ⓒ $$\int f(x)\,dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^{n+1}}{n5^n(n+1)}$$

$\underline{x=0}$: $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5)^{n+1}}{n5^n(n+1)} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5)^n(-5)^1}{n(5^n)(n+1)}$$

$$\sum_{n=1}^{\infty} \frac{\overbrace{(-1)^{n+1}(-1)^n}^{-1}(-5)}{n(n+1)} = \sum_{n=1}^{\infty} \frac{5}{n(n+1)}$$

Telescoping → convg.

$\underline{x=10}$: $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(5)^{n+1}}{n5^n(n+1)} \to \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cancel{(5)^n}(5)^1}{n\cancel{5^n}(n+1)}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(5)}{n(n+1)}$$

Alt. Series Test

$$\lim_{n\to\infty} \frac{5}{n(n+1)} = 0 \checkmark$$

$$\frac{5}{(n+1)(n+2)} \le \frac{5}{n(n+1)} \checkmark$$

converg.

$$\boxed{[0,10]}$$

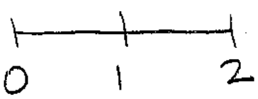
③ $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$ $c = 1$

ⓐ

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+2}}(x-1)^{n+2}}{n+2} \cdot \frac{(n+1)}{\cancel{(-1)^{n+1}}\cancel{(x-1)^{n+1}}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(x-1)(n+1)}{(n+2)} \right| = |x-1| \lim_{n\to\infty} \left| \frac{n+1}{n+2} \right| = |x-1|$$

$$|x-1| < 1 \quad \boxed{R = 1}$$



$\underline{x=0}$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{n+1} = \sum_{n=1}^{\infty} \frac{(1)^{n+1}}{n+1}$

$$\sum_{n=1}^{\infty} \frac{1}{n+1} \qquad a_n = \frac{1}{n+1} \quad b_n = \frac{1}{n}$$

Limit comparison

$$\lim_{n\to\infty} \frac{1}{n} \cdot \frac{n+1}{1} = \lim_{n\to\infty} \frac{n+1}{n} = 1 \checkmark$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ p-series diverges

$\underline{x=2}$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1)^{n+1}}{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

Alt. Series Test $\lim_{n\to\infty} \frac{1}{n+1} = 0 \checkmark$

$$\frac{1}{n+2} \leq \frac{1}{n+1} \checkmark$$

$\boxed{(0, 2]}$

Converges

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1} \qquad \frac{1}{n+1}(x-1)^{n+1}$$

(b) $f'(x) = \sum_{n=1}^{\infty} (-1)^{n+1}(x-1)^n$

$\underline{x=0}$: $\sum_{n=1}^{\infty} (-1)^{n+1}(-1)^n = \sum_{n=1}^{\infty} -1$ nth term $\lim_{n\to\infty} -1 = -1 \neq 0$

diverges

$\underline{x=2}$: $\sum_{n=1}^{\infty} (-1)^{n+1}(1)^n = \sum_{n=1}^{\infty} (-1)^n(-1)^1(1)^n$

$\sum_{n=1}^{\infty} (-1)^n(-1)$ Alt. Series

$\lim_{n\to\infty} (-1) = -1 \neq 0$

diverg by nth term

$\boxed{(0,2)}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1} \qquad \frac{1}{n+1}(x-1)^{n+1}$$

ⓒ $$\int f(x)\,dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+2}}{(n+1)(n+2)}$$

$\underline{x=0}$: $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+2}}{(n+1)(n+2)} = \sum_{n=1}^{\infty} \frac{(-1)^n(-1)^1(-1)^n(-1)^2}{(n+1)(n+2)}$$

$$\sum_{n=1}^{\infty} \frac{-1}{(n+1)(n+2)}$$ Telescoping conv.

$\underline{x=2}$: $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1)^{n+2}}{(n+1)(n+2)} \to \sum_{n=1}^{\infty} \frac{(-1)^n(-1)^1(1)^n(1)^2}{(n+1)(n+2)}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n(-1)}{(n+1)(n+2)}$$

Alt. Series Test

$$\lim_{n\to\infty} \frac{-1}{(n+1)(n+2)} = 0 \checkmark$$

$$\frac{-1}{(n+2)(n+3)} \leq \frac{-1}{(n+1)(n+2)} \checkmark$$

converg

$$\boxed{[0,2]}$$