AP Calculus BC
Unit 10 – Sequences & Series (Part 2)

Day 3 Notes: Power Series (Part 1)

Recall that Maclaurin and Taylor polynomials are <u>finite</u> polynomials that can be used to <u>approximate</u> a function $f(x)$. For example, we found that $f(x)=e^x$ can be approximated by $P_n(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}$. The higher the degree of the polynomial, the better the approximation.We can go further, because $f(x)=e^x$ can be represented <u>**exactly**</u> by the

POWER SERIES $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots=\sum_{n=0}^{\infty}\frac{x^n}{n!}$.

<u>**Definition of power series:**</u>
Let a_n be a constant and x be a variable.
1) A power series centered at $x = 0$ takes the form

$$\sum_{n=0}^{\infty}a_nx^n=a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n+\cdots$$

2) A power series centered at $x = c$ takes the form

$$\sum_{n=0}^{\infty}a_n(x-c)^n=a_0+a_1(x-c)+a_2(x-c)^2+a_3(x-c)^3+\cdots+a_n(x-c)^n+\cdots$$

A power series $\sum_{n=0}^{\infty}a_n(x-c)^n$ is a function of *x*, and its domain is the set of all *x* for which the series converges. **A series will always converge at its center!**

<u>**Convergence of a power series centered at x = c**</u>
One of the following will be true:

1. The series converges only at $x = c$. This means that the radius of convergence is 0. ($R = 0$)

2. There exists a number *R*, $R > 0$, such that

   $|x-c|<R \Rightarrow$ the series converges
   $|x-c|>R \Rightarrow$ the series diverges

   - *R* is called the <u>**radius of convergence**</u>.
   - The set of all *x* for which the power series converges is called the <u>**interval of convergence**</u>.

3. The series converges absolutely for all x. This means the radius of convergence is ∞.

We use the **Ratio Test** to determine the radius of convergence *R*.

Examples:

1) At what point is each series centered?

a. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

b. $\sum_{n=0}^{\infty} 3(x+4)^n$

2) Find the radius of convergence (*R*).

a. $\sum_{n=0}^{\infty} (4x)^n$

b. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$

c. $\sum_{n=0}^{\infty} n! x^n$

d. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

The interval of convergence of a power series includes the interval ($c - R$, $c + R$). **We must test the endpoints separately to determine if they should be included in the interval.**

3) Find the interval of convergence:

a. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$

b. $\displaystyle\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$

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Unit 10 – Day 3 – Assignment

#'s 1 – 3: State where the power series is centered.

1) $\sum_{n=0}^{\infty} nx^n$	**2)** $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^3}$
3) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-\pi)^{2n}}{(2n)!}$	

#'s 4 – 6: Find the radius of convergence of the power series.

4) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$	5) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$
6) $\sum_{n=0}^{\infty} \frac{(2x)^{2n}}{(2n)!}$	

#'s 7 – 12: Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

7) $\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$	8) $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n}$
9) $\sum_{n=0}^{\infty}\frac{x^n}{n!}$	10) $\sum_{n=0}^{\infty}(2n)!\left(\frac{x}{2}\right)^n$
11) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n5^n}$	12) $\sum_{n=1}^{\infty}\frac{(x-c)^{n-1}}{c^{n-1}}$