Day 3 Notes: Power Series (Part 1)

Recall that Maclaurin and Taylor polynomials are <u>finite</u> polynomials that can be used to <u>approximate</u> a function f(x). For example, we found that $f(x) = e^x$ can be approximated by

 $P_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$. The higher the degree of the polynomial, the better the

approximation. We can go further, because $f(x) = e^x$ can be represented <u>exactly</u> by the

POWER SERIES
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Definition of power series:

Let a_n be a constant and x be a variable.

1) A power series centered at x = 0 takes the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n + \dots$$

2) A power series centered at x = c takes the form

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + a_3 (x-c)^3 + \dots + a_n (x-c)^n + \dots$$

A power series $\sum_{n=0}^{\infty} a_n (x-c)^n$ is a function of x, and its domain is the set of all x for which the

series converges. A series will always converge at its center!

Convergence of a power series centered at x = c

One of the following will be true:

- 1. The series converges only at x = c. This means that the radius of convergence is 0. (R = 0)
- 2. There exists a number R, R > 0, such that

 $|x-c| < R \implies$ the series converges

 $|x-c| > R \implies$ the series diverges

- *R* is called the **radius of convergence**.
- The set of all *x* for which the power series converges is called the **interval of convergence**.
- 3. The series converges absolutely for all x. This means the radius of convergence is ∞ .

We use the **Ratio Test** to determine the radius of convergence *R*.

Examples:

1) At what point is each series centered?

a.
$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$
 b. $\sum_{n=0}^{\infty} 3(x+4)^n$

2) Find the radius of convergence (*R*).

a.
$$\sum_{n=0}^{\infty} (4x)^n$$
 b. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$

c.
$$\sum_{n=0}^{\infty} n! x^n$$
 d. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

The interval of convergence of a power series includes the interval (c - R, c + R). We must test the endpoints separately to determine if they should be included in the interval.

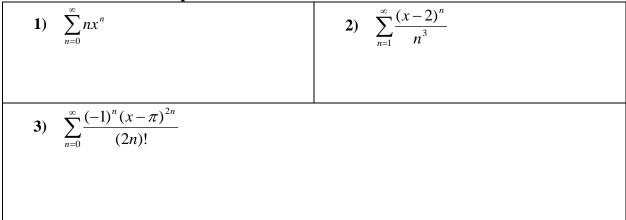
3) Find the interval of convergence:

a.
$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$$
 b. $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$

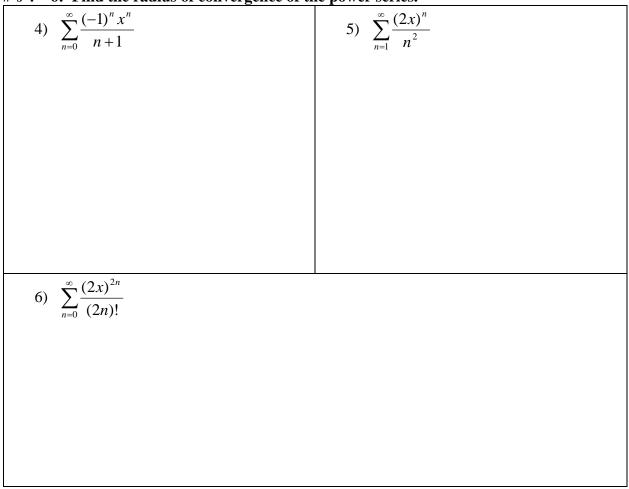
c.
$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

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#'s 1-3: State where the power series is centered.



#'s 4 – 6: Find the radius of convergence of the power series.



#'s 7 – 12: Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

for convergence at the endpoints of the interval.)	
7) $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$	$8) \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$
9) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$10) \sum_{n=0}^{\infty} (2n)! \left(\frac{x}{2}\right)^n$
$11) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-5)^n}{n5^n}$	12) $\sum_{n=1}^{\infty} \frac{(x-c)^{n-1}}{c^{n-1}}$