AP Calculus BC **Name:** Answer Key*
Unit 10 – Day 3 – Assignment

#'s 1 – 3: State where the power series is centered.

1) $\sum_{n=0}^{\infty} nx^n$ $C = 0$	**2)** $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^3}$ $C = 2$
3) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-\pi)^{2n}}{(2n)!}$ $C = \pi$	

#'s 4 – 6: Find the radius of convergence of the power series.

4) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$ $R = 1$	5) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$ $R = 1/2$
6) $\sum_{n=0}^{\infty} \frac{(2x)^{2n}}{(2n)!}$ $R = \infty$	

#'s 7 – 12: Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

7) $\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$ $(-2, 2)$	8) $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n}$ $(-1, 1]$
9) $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ $(-\infty, \infty)$	10) $\sum_{n=0}^{\infty}(2n)!\left(\frac{x}{2}\right)^n$ only converges at $x=0$
11) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n5^n}$ $(0, 10]$	12) $\sum_{n=1}^{\infty}\frac{(x-c)^{n-1}}{c^{n-1}}$, $c>0$ $(0, 2c)$

① $\sum_{n=0}^{\infty} n(x)^n$ centered at $\boxed{c=0}$

② $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^3}$ centered at $\boxed{c=2}$

③ $\sum_{n=0}^{\infty} \frac{(-1)^n (x-\pi)^{2n}}{(2n)!}$ centered at $\boxed{c=\pi}$

④ $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n+1}$ centered at $c=0$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}} x^{n+1}}{(n+2)} \cdot \frac{(n+1)}{\cancel{(-1)^n} \cancel{x^n}} \right| = \lim_{n\to\infty} \left| \frac{x(n+1)}{(n+2)} \right|$$

$$|x| \lim_{n\to\infty} \left| \frac{n+1}{n+2} \right| = |x| \cdot 1$$

$$|x| < 1 \quad \boxed{R=1}$$

⑤ $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$ centered at $c=0$

$$\lim_{n\to\infty} \left| \frac{(2x)^{n+1}}{(n+1)^2} \cdot \frac{(n^2)}{\cancel{(2x)^n}} \right| = \lim_{n\to\infty} \left| \frac{2x(n^2)}{(n+1)^2} \right|$$

$$|2x| \lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} \right| = |2x| \cdot 1$$

$$|2x| < 1 \quad |x| < \frac{1}{2} \quad \boxed{R = 1/2}$$

⑥ $\sum_{n=0}^{\infty} \frac{(2x)^{2n}}{(2n)!}$ centered at $c=0$

$$\lim_{n\to\infty} \left| \frac{(2x)^{2n+2}}{\underbrace{(2n+2)!}_{(2n+2)(2n+1)\cancel{(2n)!}}} \cdot \frac{\cancel{(2n)!}}{\cancel{(2x)^{2n}}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(2x)^2}{(2n+2)(2n+1)} \right| = |2x^2| \lim_{n\to\infty} \frac{1}{(2n+2)(2n+1)}$$

$$= |2x^2| \cdot 0 = 0$$

∴ series converges for all x.

$\boxed{R = \infty}$

⑦ $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$ centered at $c=0$

$$\lim_{n\to\infty} \left| \frac{\left(\frac{x}{2}\right)^{\cancel{n}+1}}{\cancel{\left(\frac{x}{2}\right)^n}} \right| = \lim_{n\to\infty} \left| \frac{x}{2} \right| = \left|\frac{x}{2}\right|$$

-2 0 2

$\left|\frac{x}{2}\right| < 1$ $|x| < 2$

$R=2$

$x = -2$ $\sum_{n=0}^{\infty} \left(\frac{-2}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n = -1+1+-1+1$ diverges

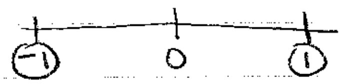
$x=2$ $\sum_{n=0}^{\infty} \left(\frac{2}{2}\right)^n = \sum_{n=0}^{\infty} (1)^n = 1+1+1+1+1$ diverges

$\boxed{(-2, 2)}$

(8) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$ centered at $c=0$

$$\lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n+1}}x^{n+1}}{(n+1)}\cdot\frac{n}{\cancel{(-1)^n}\cancel{x^n}}\right| = \lim_{n\to\infty}\left|\frac{x(n)}{(n+1)}\right|$$

$$|x|\lim_{n\to\infty}\left|\frac{n}{n+1}\right| = |x|\cdot 1$$



$|x|<1$ $R=1$

$\underline{x=-1}$ $\sum_{n=1}^{\infty}\frac{(-1)^n(-1)^n}{n} = \sum_{n=1}^{\infty}\frac{(1)^n}{n} = \sum_{n=1}^{\infty}\frac{1}{n}$ diverges (p-series)

$\underline{x=1}$ $\sum_{n=1}^{\infty}\frac{(-1)^n(1)^n}{n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ alt series test

$\lim_{n\to\infty}\frac{1}{n}=0$ ✓

$\frac{1}{n+1}\le\frac{1}{n}$ ✓

converges

$\boxed{(-1,1]}$

⑨ $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ centered at $c=0$

$$\lim_{n\to\infty} \left| \frac{x^{\cancel{n}+1}}{\underbrace{(n+1)!}_{(n+1)(\cancel{n!})}} \cdot \frac{\cancel{n!}}{\cancel{x^n}} \right| = \lim_{n\to\infty} \left| \frac{x}{n+1} \right|$$

$$|x| \lim_{n\to\infty} \left| \frac{1}{n+1} \right|$$

$$|x| \cdot 0 = 0$$

∴ Series converges for all x

$\boxed{(-\infty, \infty)}$

⑩ $\sum_{n=0}^{\infty} (2n)! \left(\frac{x}{2}\right)^n$ centered at $c=0$

$$\lim_{n\to\infty} \left| \frac{\overbrace{\cancel{(2n+2)!}}^{(2n+2)(2n+1)(\cancel{2n)!}} \left(\frac{x}{2}\right)^{\cancel{n}+1}}{\cancel{(2n)!} \cancel{\left(\frac{x}{2}\right)^n}} \right| = \lim_{n\to\infty} \left| (2n+2)(2n+1)\left(\frac{x}{2}\right) \right|$$

$$\left|\frac{x}{2}\right| \lim_{n\to\infty} |(2n+2)(2n+1)|$$

$$= \infty$$

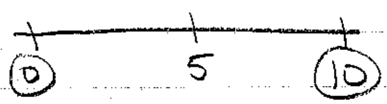
$\boxed{\therefore \text{ series only converges at } x=0}$

(11) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$ centered at $c=5$

$$\lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n+2}}(x-5)^{\cancel{n+1}}}{(n+1)5^{\cancel{n+1}}}\cdot\frac{n\cancel{5^n}}{\cancel{(-1)^{n+1}}\cancel{(x-5)^n}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(x-5)(n)}{(n+1)5}\right| = \left|\frac{x-5}{5}\right|\lim_{n\to\infty}\left|\frac{n}{n+1}\right|$$

$$= \left|\frac{x-5}{5}\right|\cdot 1$$



$$\left|\frac{x-5}{5}\right| < 1$$

$$|x-5| < 5 \quad R=5$$

$\underline{x=0}$: $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-5)^n}{n5^n}$

$\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n}{n}$ $\qquad\displaystyle\sum_{n=1}^{\infty}\frac{-1}{n}$ p-series, diverges

$\underline{x=10}$: $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cancel{(5)^n}}{n\cancel{5^n}} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$

$a_n = \frac{1}{n}$ Alt Series test

$\boxed{(0,10]}$

$\lim_{n\to\infty}\frac{1}{n} = 0$ ✓ converges

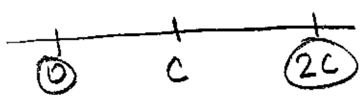
$\frac{1}{n+1} \le \frac{1}{n}$ ✓

(12) $\sum_{n=1}^{\infty} \frac{(x-c)^{n-1}}{c^{n-1}}$, $c>0$ centered at $c=0$

$$\lim_{n\to\infty}\left|\frac{\cancel{(x-c)^n}}{\cancel{c^n}}\cdot\frac{\cancel{c^{n-1}}}{\cancel{(x-c)^{n-1}}}\right|$$

$$\lim_{n\to\infty}\left|\frac{c^{-1}}{(x-c)^{-1}}\right| = \lim_{n\to\infty}\left|\frac{(x-c)}{c}\right| =$$

$$\frac{1}{c}\left|x-c\right| < 1$$



$|x-c| < c$ $R=c$

$\underline{x=0}$ $\sum_{n=1}^{\infty} \frac{(-c)^{n-1}}{c^{n-1}} = \sum_{n=1}^{\infty} (-1)^{n-1}$ diverges
Alt. Series

$\underline{x=2c}$ $\sum_{n=1}^{\infty} \frac{(c)^{n-1}}{c^{n-1}} = \sum_{n=1}^{\infty} 1$ diverges $1+1+1+1\ldots$

$$\boxed{(0, 2c)}$$