AP Calculus BC
Unit 9 – Sequences & Series (Part 2)

Day 2 Notes: Taylor's Theorem

Remainder of a Taylor Polynomial

If $f(x)$ is approximated by a Taylor polynomial $P_n(x)$, then

$$f(x) = P_n(x) + R_n(x)$$

exact approx. remainder

Therefore, the error in the approximation $P_n(x)$ is

$$\text{Error} = |R_n(x)| = |f(x) - P_n(x)|$$

TAYLOR'S THEOREM

If f has n derivatives in an interval containing $x = c$, then for each x in that interval, there is a number z, between x and c, such that

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}.$$ ← **Lagrange form of remainder**

Note: If Error = $|R_n(x)|$, then all we need is the **maximum value** of $f^{(n+1)}(z)$ on the interval from x to c. **We don't actually have to find the value of z!**

Example 1: Use Taylor's Theorem to obtain an upper bound for the error of the approximation. Then calculate the exact value of the error.

$$e \approx 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \frac{1^5}{5!}$$

Example 2: Determine the degree of the Maclaurin polynomial required for the error in the approximation of the function sin(0.75) to be less than 0.001.

Example 3: Given f(x) = cosx.

a) Write a 4^{th} degree Taylor polynomial for f(x) about x = 0.

b) Use the polynomial you found in part (a) to approximate the value of cos(0.2).

c) Use Taylor's Theorem to estimate the maximum error in your approximation.

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Unit 10 – Day 2 – Assignment

#'s 1 – 3: Find the Maclaurin polynomial of degree n for the function.

1) $f(x) = e^{2x}, \quad n = 4$	2) $f(x) = \frac{x}{x+1}, \quad n = 4$
3) $f(x) = sin\pi x, \quad n = 3$	

#'s 4 – 6: Find the nth Taylor polynomial centered at c.

4) $f(x) = \frac{2}{x^2}$, n = 4, c =2	5) $f(x) = \sqrt[3]{x}$, n = 3, c = 8

6)

$$f(x) = x^2 cosx, \qquad n = 2, c = \pi$$

#'s 7 – 8: Use Taylor's Theorem to obtain an upper bound for the error of the approximation. Then calculate the exact value of the error.

7)

$$\cos(0.3) \approx 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$$

8)

$$e \approx 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!}$$

#'s 9 – 10: Determine the degree of the Maclaurin polynomial required for the error in the approximation of the function at the indicated value of x to be less than 0.001.

9)

$$\sin(0.3)$$

10)

$$e^{0.6}$$