AP Calculus BC Name: Answer Key*
Unit 10 – Day 2 – Assignment

#'s 1 – 3: Find the Maclaurin polynomial of degree n for the function.

1) $f(x) = e^{2x}, \quad n = 4$

$P_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4$

2) $f(x) = \frac{x}{x+1}, \quad n = 4$

$P_4(x) = x - x^2 + x^3 - x^4$

3) $f(x) = \sin \pi x, \quad n = 3$

$P_3(x) = \pi x - \frac{\pi^3}{6}x^3$

#'s 4 – 6: Find the nth Taylor polynomial centered at c.

4) $f(x) = \frac{2}{x^2}$, n = 4, c = 2

$P_4(x) = \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{8}(x-2)^2 - \frac{1}{4}(x-2)^3 + \frac{5}{32}(x-2)^4$

5) $f(x) = \sqrt[3]{x}$, n = 3, c = 8

$P_3(x) = 2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20736}(x-8)^3$

6)

$$f(x) = x^2 cosx, \qquad n = 2, c = \pi$$

$P_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{(-2+\pi^2)}{2}(x-\pi)^2$

#'s 7 – 8: Use Taylor's Theorem to obtain an upper bound for the error of the approximation. Then calculate the exact value of the error.

7)	8)
$\cos(0.3) \approx 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$	$e \approx 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!}$
$R_4 \leq 2.025 \times 10^{-5}$	$R_4(x) \leq 0.0226523486$
exact → 1.0109×10^{-6}	exact → 0.009948495

#'s 9 – 10: Determine the degree of the Maclaurin polynomial required for the error in the approximation of the function at the indicated value of x to be less than 0.001.

9)	10)
$\sin(0.3)$	$e^{0.6}$
$n = 3$	$n = 5$

① $f(x) = e^{2x}$, $n=4$

$f(x) = e^{2x} \rightarrow e^{2(0)} = 1$
$f'(x) = 2e^{2x} \rightarrow 2e^{2(0)} = 2$
$f''(x) = 4e^{2x} \rightarrow 4e^{2(0)} = 4$
$f'''(x) = 8e^{2x} \rightarrow 8e^{2(0)} = 8$
$f^4(x) = 16e^{2x} \rightarrow 16e^{2(0)} = 16$

$$P_4(x) = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{{}_6\,3!} + \frac{16x^4}{{}_{24}\,4!}$$

$$\boxed{P_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4}$$

② $f(x) = \frac{x}{x+1}$, $n=4$

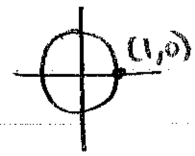
$f(x) = \frac{x}{x+1} \rightarrow \frac{0}{0+1} = 0$

$f'(x) = \frac{(x+1)(1) - (x)(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = (x+1)^{-2} \rightarrow (0+1)^{-2} = 1$

$f''(x) = -2(x+1)^{-3} \rightarrow -2(0+1)^{-3} = -2$
$f'''(x) = 6(x+1)^{-4} \rightarrow 6(0+1)^{-4} = 6$
$f^4(x) = -24(x+1)^{-5} \rightarrow -24(0+1)^{-5} = -24$

$$P_4(x) = 0 + 1x - \frac{2x^2}{2!} + \frac{6x^3}{3!} - \frac{24x^4}{4!}$$

$$\boxed{P_4(x) = x - x^2 + x^3 - x^4}$$



(3) $f(x) = \sin \pi x$, $n = 3$

$f(x) = \sin \pi x \rightarrow \sin(\pi 0) = 0$

$f'(x) = \pi \cos \pi x \rightarrow \pi \cos(\pi 0) = \pi$

$f''(x) = -\pi^2 \sin \pi x \rightarrow -\pi^2 \sin(\pi 0) = 0$

$f'''(x) = -\pi^3 \cos \pi x \rightarrow -\pi^3 \cos(\pi 0) = -\pi^3$

$$P_3(x) = \cancel{0} + \pi x + \cancel{\frac{0x^2}{2!}} - \frac{\pi^3 x^3}{3!}$$

$$\boxed{P_3(x) = \pi x - \frac{\pi^3}{6} x^3}$$

(4) $f(x) = \frac{2}{x^2}$, $n = 4$, $c = 2$

$f(x) = 2x^{-2} \rightarrow 2/x^2 \rightarrow \frac{2}{2^2} = 1/2$

$f'(x) = -4x^{-3} \rightarrow -4/x^3 \rightarrow -4/2^3 = -1/2$

$f''(x) = 12x^{-4} \rightarrow 12/x^4 \rightarrow 12/2^4 = 12/16 = 3/4$

$f'''(x) = -48x^{-5} \rightarrow -48/x^5 \rightarrow -48/2^5 = -48/36 = -3/2$

$f^{4}(x) = 240x^{-6} \rightarrow 240/x^6 \rightarrow 240/2^6 = 15/4$

$$P_4(x) = \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{4}\left(\frac{1}{2!}\right)(x-2)^2 - \frac{3}{2}\left(\frac{1}{3!}\right)(x-2)^3 + \frac{15}{4}\left(\frac{1}{4!}\right)(x-2)^4$$

$$\boxed{P_4(x) = \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{8}(x-2)^2 - \frac{1}{4}(x-2)^3 + \frac{5}{32}(x-2)^4}$$

⑤ $f(x) = \sqrt[3]{x}$, $n=3$, $c=8$

$f(x) = x^{1/3} \rightarrow \sqrt[3]{x} \rightarrow \sqrt[3]{8} = 2$

$f'(x) = \frac{1}{3}x^{-2/3} \rightarrow \frac{1}{3\sqrt[3]{x^2}} \rightarrow \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$

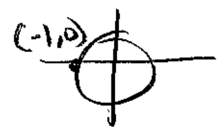
$f''(x) = -2/9x^{-5/3} \rightarrow \frac{-2}{9\sqrt[3]{x^5}} \rightarrow \frac{-2}{9\sqrt[3]{8^5}} = -1/144$

$f'''(x) = 10/27x^{-8/3} \rightarrow \frac{10}{27\sqrt[3]{x^8}} \rightarrow \frac{10}{27\sqrt[3]{8^8}} = \frac{5}{3456}$

$P_3(x) = 2 + \frac{1}{12}(x-8) - \frac{1}{144}\left(\frac{1}{2!}\right)(x-8)^2 + \frac{5}{3456}\left(\frac{1}{3!}\right)(x-8)^3$

$$\boxed{P_3(x) = 2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20736}(x-8)^3}$$

⑥ $f(x) = x^2\cos x$, $n=2$, $c=\pi$



$f(x) = x^2\cos x \rightarrow \pi^2\cos\pi = -\pi^2$

$f'(x) = (2x)(\cos x) + (x^2)(-\sin x) = 2\pi\cos\pi + \pi^2\cancel{(-\sin\pi)} = -2\pi$

$f''(x) = (2)(\cos x) + (2x)(-\sin x) + (2x)(-\sin x) + (x^2)(-\cos x)$

$2\cos\pi + 2\pi\cancel{(-\sin\pi)} + 2\pi\cancel{(-\sin\pi)} + \pi^2(-\cos\pi)$

$-2 + \pi^2$

$$\boxed{P_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{(-2+\pi^2)}{2}(x-\pi)^2}$$

(7) $\cos(0.3) \approx 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$ $c=0$ $P_4(x)$

$f(x) = \cos x$
$f'(x) = -\sin x$
$f''(x) = -\cos x$
$f'''(x) = \sin x$
$f^4(x) = \cos x$
$f^5(x) = -\sin x$

$$R_4(x) = \frac{f^5(z)}{5!}(x-0)^5$$

max of $f^5(x) = -\sin x$ on $[0, 0.3] = 1$

$$R_4(x) = \frac{1}{5!}(0.3)^5$$

$$R_4(x) = 2.025 \times 10^{-5}$$

$$\boxed{R_4(x) \le 2.025 \times 10^{-5}}$$

exact error →

$\cos(0.3) = 0.9553364891$

$1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!} = 0.9553375$

$.9553375 - 0.9553364891 = \boxed{1.0109 \times 10^{-6}}$

(8) $e^1 \approx 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!}$ $P_4(x)$, $c = 0$

$$R_4(x) = \frac{f^5(z)}{5!}(x-0)^5$$

max of e^x on $[0,1] = e^1$

$= \frac{e}{5!}(1)^5 = 0.0226523486$

$$\boxed{R_4(x) \le 0.0226523486}$$

exact error →

$e^1 = 2.718281828$

$1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} = 2.708\overline{3}33333$

$\boxed{\text{exact error} = 0.009948495}$

(9) $\sin(0.3)$

$f(x) = \sin x$

max of sinx

$$R_n(x) \le \frac{1}{(n+1)!}(0.3)^{n+1} < 0.001$$

$\underline{n=1}$ $\frac{1}{(1+1)!}(0.3)^{1+1} = .045$

$\underline{n=2}$ $\frac{1}{(2+1)!}(0.3)^{2+1} = 0.0045$

$\underline{n=3}$ $\frac{1}{(3+1)!}(0.3)^{3+1} = 0.0003375$ ✓ $\boxed{n=3}$

(10) $e^{0.6}$

$$R_n(x) \le \frac{e^{0.6}}{(n+1)!}(0.6)^{n+1} < 0.001$$

$\underline{n=3}$ $\frac{e^{.6}}{(3+1)!}(0.6)^{3+1} = .00984$

$\underline{n=4}$ $\frac{e^{.6}}{(4+1)!}(0.6)^{4+1} = .00118$

$\underline{n=5}$ $\frac{e^{.6}}{(5+1)!}(0.6)^{5+1} = .000118$ ✓

$\boxed{n=5}$