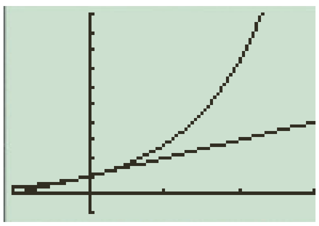
AP Calculus BC
Unit 9 – Sequences & Series (Part 2)

Day 1 Notes: Maclaurin & Taylor Polynomials

Recall: We can use the equation of the tangent to a curve at $x = c$ to approximate the value of a function for some x near c.

For example, let $f(x) = e^x$. Approximate the value of $e^{0..1}$ using the tangent line at $x = 0$.



Polynomial approximation uses this logic: If linear approximation is relatively accurate for values of x close to $x = c$, maybe a quadratic approximation would be better...or maybe a cubic would be even better still... and so on. In fact, polynomial approximations usually give us a larger interval on which the approximation is relatively accurate.

Maclaurin polynomial:

If f has n derivatives at $c = 0$, then

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

is the nth degree polynomial approximation of f at $c = 0$.

<u>Example 1</u>: Find the Maclaurin polynomial of degree 4 for $f(x) = e^x$. Then use the function to approximate f(0.1).

Example 2: Find a third degree Maclaurin polynomial for $f(x) = \sin(x)$.

Taylor polynomial:

TAYLOR POLYNOMIAL

If f has n derivatives at $x = c$, then the nth Taylor polynomial approximation of f at $x = c$ is

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

Example 3: Find $P_2(x)$ for $f(x) = \sqrt{x}$ at $c = 4$.

Example 4: Find a second degree Taylor polynomial for $f(x) = x^2 \cos x$ centered at π.

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Unit 10 – Day 1 – Assignment

#'s 1 – 4: Find the Maclaurin polynomial of degree n for the function.

1) $f(x) = e^{-x}, \quad n = 3$	2) $f(x) = xe^{x}, \quad n = 4$
3) $f(x) = \frac{1}{x+1}, \quad n = 4$	4) $f(x) = secx, \quad n = 2$

#'s 5 - 7: Find the nth Taylor polynomial centered at c.

5) $f(x) = \frac{1}{x}, \quad n = 4, c = 1$	6) $f(x) = \sqrt{x}, \quad n = 4, c = 1$
7) $f(x) = lnx, \quad n = 4, c = 1$	

#'s 8 – 9: Approximate the function at the given value of x, using the polynomial found in the indicated previous problem.

8) $f(x) = e^{-x}, \; f\left(\frac{1}{2}\right), \; problem\ \#1$	9) $f(x) = lnx, f(1.2), problem\ \#7$