AP Calculus BC
Unit 10 – Day 1 – Assignment

Name: Answer Key*

#'s 1 – 4: Find the Maclaurin polynomial of degree n for the function.

1) $f(x) = e^{-x}, \quad n = 3$ $\boxed{P_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3}$	2) $f(x) = xe^x, \quad n = 4$ $\boxed{P_4(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4}$
3) $f(x) = \frac{1}{x+1}, \quad n = 4$ $\boxed{P_4 = 1 - x + x^2 - x^3 + x^4}$	4) $f(x) = secx, \quad n = 2$ $\boxed{P_2(x) = 1 + \frac{1}{2}x^2}$

#'s 5 - 7: Find the nth Taylor polynomial centered at c.

5)

$$f(x) = \frac{1}{x}, \qquad n = 4, c = 1$$

$$\boxed{P_4(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4}$$

6)

$$f(x) = \sqrt{x}, \qquad n = 4, c = 1$$

$$\boxed{P_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4}$$

7)

$$f(x) = lnx, \qquad n = 4, c = 1$$

$$\boxed{P_4(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4}$$

#'s 8 – 9: Approximate the function at the given value of x, using the polynomial found in the indicated previous problem.

8)

$$f(x) = e^{-x}, \; f\left(\frac{1}{2}\right), \; problem \; \#1$$

$$\boxed{0.6042}$$

9)

$$f(x) = lnx, f(1.2), problem \; \#7$$

$$\boxed{0.1823}$$

① Maclaurin polynomial

$f(x) = e^{-x}$, $n = 3$, $c = 0$

$$f(x) = e^{-x} \qquad f(0) = e^{-0} = 1$$
$$f'(x) = -e^{-x} \qquad f'(0) = -e^{-0} = -1$$
$$f''(x) = e^{-x} \qquad f''(0) = e^{-0} = 1$$
$$f'''(x) = -e^{-x} \qquad f'''(0) = -e^{-0} = -1$$

$$P_3(x) = 1 - 1x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3$$

$$\boxed{P_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3}$$

② Maclaurin polynomial

$f(x) = xe^{x}$, $n = 4$, $c = 0$

$$f(x) = xe^{x} \qquad f(0) = (0)(e^0) = 0$$
$$f'(x) = (1)(e^x) + (x)(e^x) = e^x + xe^x \qquad f'(0) = e^0 + (0)e^0 = 1$$
$$f''(x) = e^x + e^x + xe^x = 2e^x + xe^x \qquad f''(0) = 2e^0 + (0)e^0 = 2$$
$$f'''(x) = 2e^x + e^x + xe^x = 3e^x + xe^x \qquad f'''(0) = 3e^0 + (0)e^0 = 3$$
$$f^{(4)}(x) = 3e^x + e^x + xe^x = 4e^x + xe^x \qquad f^4(x) = 4e^0 + (0)e^0 = 4$$

$$P_4(x) = 0 + 1x + \frac{2}{2!}x^2 + \frac{3}{3!}x^3 + \frac{4}{4!}x^4$$

$3 \cdot 2$ $\qquad$ $4 \cdot 3 \cdot 2$

$$\boxed{P_4(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4}$$

③ Maclaurin polynomial

$f(x) = \frac{1}{x+1}$, $n=4$, $C=0$

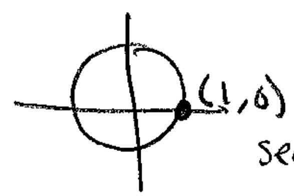
$$f(x) = (x+1)^{-1} = \frac{1}{x+1} \qquad f(0) = \frac{1}{0+1} = 1$$
$$f'(x) = -1(x+1)^{-2} = \frac{-1}{(x+1)^2} \qquad f'(0) = \frac{-1}{(0+1)^2} = -1$$
$$f''(x) = 2(x+1)^{-3} = \frac{2}{(x+1)^3} \qquad f''(0) = \frac{2}{(0+1)^3} = 2$$
$$f'''(x) = -6(x+1)^{-4} = \frac{-6}{(x+1)^4} \qquad f'''(0) = \frac{-6}{(0+1)^4} = -6$$
$$f^4(x) = 24(x+1)^{-5} = \frac{24}{(x+1)^5} \qquad f^4(0) = \frac{24}{(0+1)^5} = 24$$

$$P_4(x) = 1 - 1x + \frac{2}{2!}x^2 - \frac{6}{\underset{3\cdot 2}{3!}}x^3 + \frac{24}{\underset{4\cdot 3\cdot 2}{4!}}x^4$$

$$\boxed{P_4(x) = 1 - x + x^2 - x^3 + x^4}$$

④ Maclaurin polynomial

$f(x) = \sec x$, $n=2$, $C=0$



$\tan = \frac{x}{y}$

$\sec = \frac{1}{x}$

$$f(x) = \sec x$$
$$f'(x) = \sec x \tan x$$
$$f''(x) = (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)$$
$$= \sec x \tan^2 x + \sec^3 x$$

$$f(0) = \sec(0) = \frac{1}{1} = 1$$
$$f'(0) = \sec(0)\tan(0) = \left(\frac{1}{1}\right)\left(\frac{0}{1}\right) = 0$$
$$f''(0) = \sec(0)\tan^2(0) + \sec^3(0)$$
$$\left(\frac{1}{1}\right)\left(\frac{0}{1}\right)^2 + \left(\frac{1}{1}\right)^3 = 1$$

$$P_2(x) = 1 + 0x + \frac{1}{2!}x^2$$

$$\boxed{P_2(x) = 1 + \frac{1}{2}x^2}$$

(5) Taylor Polynomial

$f(x) = \frac{1}{x}$, $n=4$, $c=1$

$f(x) = x^{-1} = \frac{1}{x}$ $\quad f(1) = \frac{1}{1} = 1$

$f'(x) = -1x^{-2} = \frac{-1}{x^2}$ $\quad f'(1) = \frac{-1}{1^2} = -1$

$f''(x) = 2x^{-3} = \frac{2}{x^3}$ $\quad f''(1) = \frac{2}{1^3} = 2$

$f'''(x) = -6x^{-4} = \frac{-6}{x^4}$ $\quad f'''(1) = -\frac{6}{1^4} = -6$

$f^4(x) = 24x^{-5} = \frac{24}{x^5}$ $\quad f^4(1) = \frac{24}{1^5} = 24$

$$P_4(x) = 1 - 1(x-1) + \frac{2}{2!}(x-1)^2 - \frac{6}{\underset{3\cdot 2}{3!}}(x-1)^3 + \frac{24}{\underset{4\cdot 3\cdot 2}{4!}}(x-1)^4$$

$$\boxed{P_4(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4}$$

(6) Taylor Polynomial

$f(x) = \sqrt{x}$, $n=4$, $c=1$

$f(x) = x^{1/2}$ $\quad f(1) = (1)^{1/2} = 1$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(1) = \frac{1}{2}(1)^{-1/2} = \frac{1}{2}$

$-\frac{1}{4}\cdot\frac{-3}{2}$ $\quad f''(x) = -\frac{1}{4}x^{-3/2}$ $\quad f''(1) = -\frac{1}{4}(1)^{-3/2} = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}x^{-5/2}$ $\quad f'''(1) = \frac{3}{8}(1)^{-5/2} = 3/8$

$\frac{3}{8}\cdot\frac{-5}{2}$ $\quad f^4(x) = \frac{-15}{16}x^{-7/2}$ $\quad f^4(1) = -15/16\,(1)^{-7/2} = -15/16$

$$P_4(x) = 1 + \frac{1}{2}(x-1) - \underset{-\frac{1}{4}\cdot\frac{1}{2}}{\frac{1}{4}\left(\frac{1}{2!}\right)}(x-1)^2 + \underset{\frac{3}{8}\cdot\frac{1}{6}}{\frac{3}{8}\left(\frac{1}{3!}\right)}(x-1)^3 - \underset{-\frac{15}{16}\cdot\frac{1}{24}}{\frac{15}{16}\left(\frac{1}{4!}\right)}(x-1)^4$$

$$\boxed{P_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4}$$

7. Taylor polynomial

$f(x) = \ln x, \quad n=4, \quad c=1$

$f(x) = \ln x \qquad f(1) = \ln(1) = 0$

$f'(x) = \frac{1}{x} = x^{-1} \qquad f'(1) = \frac{1}{1} = 1$

$f''(x) = -1x^{-2} = -\frac{1}{x^2} \qquad f''(1) = -\frac{1}{1^2} = -1$

$f'''(x) = 2x^{-3} = \frac{2}{x^3} \qquad f'''(1) = \frac{2}{1^3} = 2$

$f^4(x) = -6x^{-4} = -\frac{6}{x^4} \qquad f^4(1) = -\frac{6}{1^4} = -6$

$$P_4(x) = 0 + 1(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{\underset{3\cdot 2}{3!}}(x-1)^3 - \frac{6}{\underset{4\cdot 3\cdot 2}{4!}}(x-1)^4$$

$$\boxed{P_4(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4}$$

8. Approximate $\rightarrow f(x) = e^{-x}$, $f(\frac{1}{2})$, using #1

$f(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$

$f(\frac{1}{2}) = 1 - \frac{1}{2} + \frac{1}{2}(\frac{1}{2})^2 - \frac{1}{6}(\frac{1}{2})^3$

$\approx \boxed{0.6042}$

9. Approximate $\rightarrow f(x) = \ln x$, $f(1.2)$, using #7

$f(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$

$f(1.2) = (1.2-1) - \frac{1}{2}(1.2-1)^2 + \frac{1}{3}(1.2-1)^3 - \frac{1}{4}(1.2-1)^4$

$\approx \boxed{0.1823}$