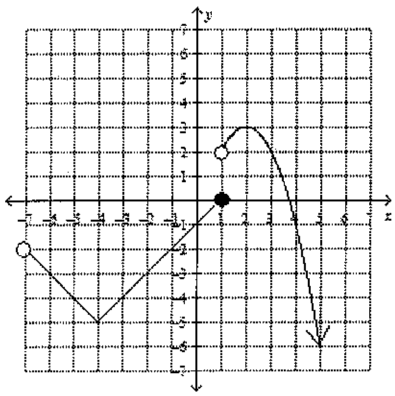
AP Calculus AB Name: Answer Key*

Unit 1 – Day 4 – Free Response Practice

Calculator NOT Permitted



Graph of $g(x)$

$$f(x)=\begin{cases} ax+3, & x<-3 \\ x^2-3x, & -3\le x<2 \\ bx-5, & x\ge 2 \end{cases}$$

Equation of $f(x)$

Pictured above is the graph of a function $g(x)$ and the equation of a piece-wise defined function $f(x)$. Answer the following questions.

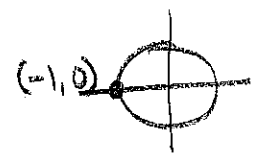
a. Find $\lim_{x\to1^+}\left[2g(x)-f(x)\cdot\cos\pi x\right]$. Show your work applying the properties of limits.

$$2\cdot\lim_{x\to1^+} g(x) - \lim_{x\to1^+} f(x)\cdot\lim_{x\to1^+}\cos\pi x$$

$$2\,(2) - \left[(1)^2-(3(1))\right]\cdot\cos(\pi(1))$$

$$4 - [-2](-1)$$

$$= 4 - 2 = \boxed{2}$$



b. On its domain, what is one value of x at which $g(x)$ is discontinuous? Use the three part definition of continuity to explain why $g(x)$ is discontinuous at this value.

The only value of x on the domain of $g(x)$ at which $g(x)$ is discontinuous is at $x=1$.

I. $g(1)=0$, $g(1)$ is defined. ✓

II. $\lim_{x\to1^-} g(x)=0$, $\lim_{x\to1^+} g(x)=2$, so $\lim_{x\to1} g(x)$ D.N.E.

$\therefore$ $g(x)$ is not continuous at $x=1$, since $\lim_{x\to1} g(x)$ D.N.E.

c. For what value(s) of *a* and *b*, if they exist, would the function *f*(*x*) be continuous everywhere? Justify your answer using limits.

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x)$$

$$\downarrow$$

$$a(-3) + 3 = (-3)^2 - 3(-3)$$

$$-3a + 3 = 9 + 9$$

$$-3a + 3 = 18$$

$$-3a = 15$$

$$\boxed{a = -5}$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\downarrow \qquad \downarrow$$

$$(2)^2 - 3(2) = b(2) - 5$$

$$4 - 6 = 2b - 5$$

$$-2 = 2b - 5$$

$$3 = 2b$$

$$\boxed{b = 3/2}$$