Answer Key*

PARAMETRIC & POLAR REVIEW

NO CALCULATORS

1. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?

(A) $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$

2. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

3. Which of the following gives the length of the path described by the parametric equations $x(t) = 2 + 3t$ and $y(t) = 1 + t^2$ from $t = 0$ to $t = 1$?

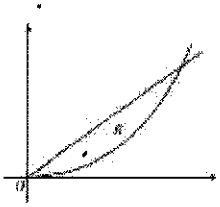
(A) $\int_0^1 \sqrt{1 + \frac{4t^2}{9}}\,dt$

(B) $\int_0^1 \sqrt{1 + 4t^2}\,dt$

(C) $\int_0^1 \sqrt{3 + 3t + t^2}\,dt$

(D) $\int_0^1 \sqrt{9 + 4t^2}\,dt$

(E) $\int_0^1 \sqrt{(2 + 3t)^2 + (1 + t^2)^2}\,dt$

4. 

Let R be the region in the first quadrant that is bounded by the polar curves $r = \theta$ and below by the line $\theta = k$. What is the area of R in terms of k?

(A) $\frac{k^3}{6}$ (B) $\frac{k^3}{3}$ (C) $\frac{k^3}{2}$ (D) $\frac{k^2}{4}$ (E) $\frac{k^2}{2}$

5. What is the slope of the line tangent to the polar curve $r = \cos\theta$ at the point where $\theta = \frac{\pi}{6}$?

(A) $-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

6. A particle moves in the xy-plane with position given by $(x(t), y(t)) = (5 - 2t, t^2 - 3)$ at time t. In which direction is the particle moving as it passes through the point $(3, -2)$?

(A) Up and to the left

(B) Down and to the left

(C) Up and to the right

(D) Down and to the right

(E) Straight up

7. Which of the following integrals gives the area of the region that is bounded by the graphs of the polar equations $\theta = 0$, $\theta = \frac{\pi}{4}$, and $r = \dfrac{2}{\cos\theta + \sin\theta}$?

(A) $\displaystyle\int_0^{\pi/4} \frac{1}{\cos\theta + \sin\theta}\,d\theta$

(B) $\displaystyle\int_0^{\pi/4} \frac{2}{\cos\theta + \sin\theta}\,d\theta$

(C) $\displaystyle\int_0^{\pi/4} \frac{2}{(\cos\theta + \sin\theta)^2}\,d\theta$

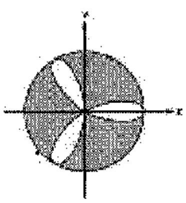
(D) $\displaystyle\int_0^{\pi/4} \frac{4}{(\cos\theta + \sin\theta)^2}\,d\theta$

(E) $\displaystyle\int_0^{\pi/4} \frac{2(\cos\theta - \sin\theta)^2}{(\cos\theta + \sin\theta)^4}\,d\theta$

8. If $x(t) = t^2 + 4$ and $y(t) = t^4 + 3$, for $t > 0$, then in terms of t, $\dfrac{d^2y}{dx^2} =$

(A) $\frac{1}{2}$ (B) 2 (C) $4t$ (D) $6t^2$ (E) $12t^2$

CALCULATOR ACTIVE

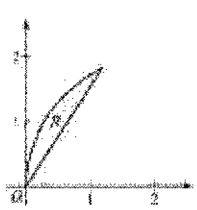
9. 

The figure shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

10. The position of a particle moving in the xy-plane is given by the parametric functions $x(t)$ and $y(t)$ for which $x'(t) = t\sin t$ and $y'(t) = 5e^{-3t} + 2$. What is the slope of the line tangent to the path of the particle at the point at which $t = 2$?

(A) 0.904 (B) 1.107 (C) 1.819 (D) 2.012 (E) 3.660

11. 

Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$. What is the area of R?

(A) 0.317 (B) 0.465 (C) 0.929 (D) 2.618 (E) 5.819

12. For $t \geq 0$, the components of the velocity of a particle moving in the xy-plane are given by the parametric equations $x'(t) = \dfrac{1}{t+1}$ and $y'(t) = ke^{kt}$, where k is a positive constant. The line $y = 4x + 3$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k?

(A) 0.072 (B) 0.433 (C) 0.495 (D) 0.803 (E) 0.828

① $x(t) = t^3 - 3t^2$ $\quad y(t) = 12t - 3t^2$

particle at rest
when $v(t) = 0$

$v(t) = \langle 3t^2 - 6t,\ 12 - 6t \rangle$

$3t^2 - 6t = 0$ $\qquad 12 - 6t = 0$

$3t(t-2) = 0$ $\qquad 12 = 6t$

$t = 0,\ t = 2$ $\qquad t = 2$

at $t = 2$, particle at rest.

$x(2) = 2^3 - 3(2)^2 = 8 - 12 = -4$ $\qquad \boxed{(-4, 12)}$

$y(2) = 12(2) - 3(2)^2 = 24 - 12 = 12$

② $r = 1 + 2\sin\theta$ at $\theta = 0$, slope = ?

$x = r\cos\theta$ $\qquad y = r\sin\theta$

$x = (1 + 2\sin\theta)(\cos\theta)$ $\qquad y = (1 + 2\sin\theta)(\sin\theta)$

$\sin 0 = 0$

$\cos 0 = 1$

$$\frac{dy}{dx} = \frac{(2\cos\theta)(\sin\theta) + (1 + 2\sin\theta)(\cos\theta)}{(2\cos\theta)(\cos\theta) + (1 + 2\sin\theta)(-\sin\theta)}$$

$$= \frac{(2)(0) + (1+0)(1)}{(2)(1) + (1+0)(-0)}$$

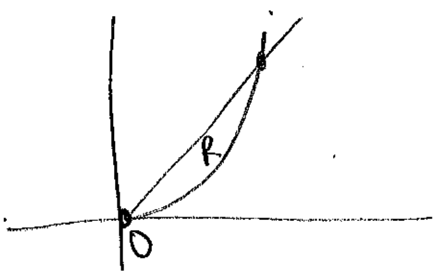
$$= \frac{0 + 1}{2 + 0} = \boxed{\frac{1}{2}}$$

③ Length $x(t) = 2+3t \quad y(t) = 1+t^2$ from $t=0$ to $t=1$

$$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$= \int_0^1 \sqrt{(3)^2 + (2t)^2}\, dt = \boxed{\int_0^1 \sqrt{9+4t^2}\, dt}$$

④



area? $r=\theta$ and $\theta = K$

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta$$

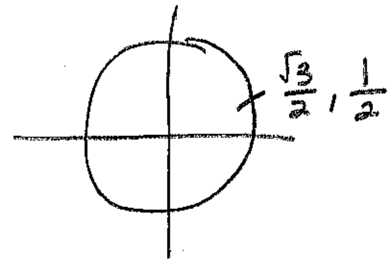
$$= \frac{1}{2}\int_0^K \theta^2\, d\theta$$

$$= \frac{1}{2}\left[\frac{1}{3}\theta^3\right]_0^k$$

$$= \frac{1}{2}\left[\frac{1}{3}k^3 - \frac{1}{3}\cancel{(0)^3}\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}k^3\right] = \boxed{\frac{k^3}{6}}$$

(5) slope $r = \cos\theta$ at $\theta = \pi/6$.



$x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$x = (\cos\theta)(\cos\theta)$ $\quad$ $y = (\cos\theta)(\sin\theta)$

$\sin(\pi/6) = \frac{1}{2}$

$\cos(\pi/6) = \frac{\sqrt{3}}{2}$

$$\frac{dy}{dx} = \frac{(-\sin\theta)(\sin\theta) + (\cos\theta)(\cos\theta)}{(-\sin\theta)(\cos\theta) + (\cos\theta)(-\sin\theta)}$$

$$= \frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)}$$

$$= \frac{-\frac{1}{4} + \frac{3}{4}}{-\frac{\sqrt{3}}{4} + -\frac{\sqrt{3}}{4}} = \frac{\frac{2}{4}}{\frac{-2\sqrt{3}}{4}} = \frac{\cancel{2}}{\cancel{4}} \cdot \frac{\cancel{4}}{-\cancel{2}\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

(6) $(5-2t, t^2-3)$ what direction as moves through pt. $(3, -2)$.

$\quad x(t) \quad y(t)$

$x'(t) = -2 \rightarrow x'(1) = -2$ Left

$y'(t) = 2t \rightarrow y'(\cancel{-2}) = 2(1) = 2$ up

$5 - 2t = 3$ $\quad$ $t^2 - 3 = -2$

$-2t = -2$ $\quad$ $t^2 = 1$

$t = 1$ $\quad$ $t = 1$

$t = 1$

(7) Area $\theta = 0, \theta = \pi/4, \quad r = \dfrac{2}{\cos\theta + \sin\theta}$

$$\text{Area} = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} \left(\frac{2}{\cos\theta + \sin\theta}\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{4}{(\cos\theta + \sin\theta)^2} \, d\theta = \boxed{\int_0^{\pi/4} \frac{2}{(\cos\theta + \sin\theta)^2} \, d\theta}$$

(8) $x(t) = t^2 + 4 \quad y(t) = t^4 + 3 \quad t > 0$

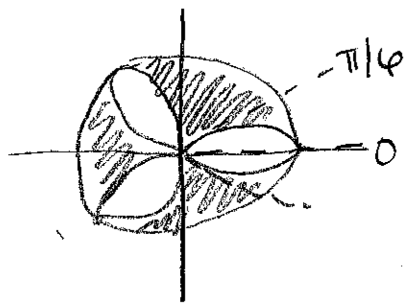
$$\frac{dy}{dx} = \frac{4t^3}{2t} = 2t^2$$

deriv.

$$\frac{d^2y}{dx^2} = \frac{4t}{2t} = \boxed{2}$$

(9) $r = 2\cos(3\theta)$ $\quad r = 2$

Sum of areas of shaded region



Area of circle $= \pi r^2 = \pi(2)^2 = 4\pi$

Area of rose curve =

$$\underbrace{\frac{1}{2}\int_{0}^{\pi/6} (2\cos 3\theta)^2 \, d\theta}_{\text{math 9}} = 0.524$$

$2\cos 3\theta = 0$

$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}$

$\theta = \pi/6$

$0.524(2) = \underbrace{1.047}_{\text{1 petal}}$

3 petals $= 1.047(3) = 3.142$

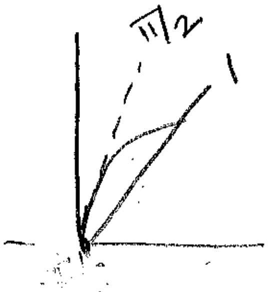
Area Shaded $= 4\pi - 3.142 = \boxed{9.424}$

(10) $x'(t) = t\sin t \quad y'(t) = 5e^{-3t} + 2$

Slope of tangent line at $t = 2$.

$$\frac{dy}{dx} = \frac{5e^{-3(2)} + 2}{2\sin 2} = \frac{2.012}{1.819} = \boxed{1.106}$$

(11)



$r = 4\cos\theta \quad \theta = 1 \quad \text{Area} = ?$

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta$$

$4\cos\theta = 0$
$\cos\theta = 0$
$\theta = \pi/2$

$$\frac{1}{2}\int_{1}^{\pi/2} (4\cos\theta)^2 \, d\theta \quad \text{math 9}$$

$$= \boxed{0.465}$$

(12) $x'(t) = \dfrac{1}{t+1} \qquad y'(t) = ke^{kt}$

$y = 4x + 3$ is parallel to tangent line at $t = 2$

↑

slope $= 4$

$$\frac{dy}{dx} = \frac{ke^{kt}}{\frac{1}{t+1}} = 4$$

$$= \frac{ke^{2k}}{\frac{1}{2+1}} = 4$$

$$\underbrace{ke^{2k}}_{y_1} = \underbrace{\frac{4}{3}}_{y_2}$$

2nd trace intersect

$$\boxed{k = 0.495}$$