

**AP Calculus BC  
Midterm Review**

Name: \_\_\_\_\_

1. What are all of the horizontal asymptotes of all the solutions of the logistics differential equation  $\frac{dy}{dx} = y(16 - 2y)$ ?

2.  $\int \sec^5 x \tan^3 x dx =$

3. Evaluate the integral:  $\int e^x \cos x dx$

4. What is the carrying capacity for a population whose growth rate is modeled by  $\frac{dP}{dt} = 45P - 9P^2$ ?

5. Evaluate the integral:  $\int \frac{5}{x^2+8x+18} dx$

6. Evaluate the integral:  $\int \sin^3(x) \cos^2(x) dx$

7.  $\int -5x \cos 2x dx$

8.  $\int \frac{4x-1}{x^2-3x-40} dx$

9.  $\int x \cos(2\pi x^2) dx$

10. Evaluate the integral:  $\int x\sqrt{x+1} dx$

11. Evaluate the integral:  $\int 3x(x^2 - 1)^4 dx$

12.  $\int \frac{5}{\sqrt{1-16x^2}} dx$

13. Which of the following integrals are divergent?

I.  $\int_2^{\infty} \frac{x}{(1+x^2)^2} dx$

II.  $\int_1^{\infty} \frac{1}{x} dx$

III.  $\int_2^{\infty} \cos 2x dx$

14. What is the value of  $\sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n$  ?

15. Which of the following series converges?

I.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

II.  $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

III.  $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

16. Determine whether the following sequence converges or diverges. If it converges, find its limit.

$$\left\{ \frac{(n-2)!}{(n+1)!} \right\}, n = 0, 1, 2, \dots$$

17. Investigate  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$  for convergence or divergence.

18. Find the third term of the sequence  $\left\{ \frac{(-1)^n (2^n + 1)}{n!} \right\}, n = 1, 2, 3, \dots$

19. Determine whether the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$  is convergent or divergent. If convergent, classify the series as absolutely convergent or conditionally convergent.

20. Find the number of terms necessary to approximate the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$  with an error of less than or equal to 0.001.

21. Determine if the following sequence converges or diverges. If it converges, find its limit.

$$\left\{ \frac{5n-1}{3n+1} \right\}, n = 1, 2, 3, \dots$$

22. Determine which series diverges.

a)  $\sum_{n=0}^{\infty} \frac{n!}{6n!-1}$

b)  $\sum_{n=1}^{\infty} \frac{1}{n^6}$

c)  $\sum_{n=0}^{\infty} 5\left(\frac{1}{10}\right)^n$

d)  $\sum_{n=0}^{\infty} \frac{n}{2^n}$

23. Determine if the series converges or diverges.  $\sum_{n=1}^{\infty} \frac{3}{(2n-1)(2n+1)}$ .

24. Determine if the series converges or diverges.  $\sum_{n=1}^{\infty} \left( \frac{2n-1}{3n+5} \right)^n$

25. Which of the following series converge?

I.  $\sum_{n=1}^{\infty} \frac{n}{n+5}$       II.  $\sum_{n=1}^{\infty} \frac{1}{n-3}$       III.  $\sum_{n=1}^{\infty} \frac{1}{n}$

26. What is the radius of convergence for the power series  $\sum_{n=0}^{\infty} \frac{(x-5)^n}{2 \cdot 3^{n+1}}$ ?

27. Find the interval of convergence for a power series that is centered at -2 for the function  $f(x) = \frac{3}{2-4x}$ .

28. Let  $f$  be the function given by  $f(x) = \ln(3-x)$ . The third-degree Taylor polynomial for  $f$  about  $x = 2$  is

29. Write out the first four terms of the Taylor series for  $f(x) = x \cos x$  about  $x = 0$ .

30. The third-degree Taylor polynomial for a function  $f$  about  $x = 4$  is

$$\frac{(x-4)^3}{512} - \frac{(x-4)^2}{64} + \frac{(x-4)}{4} + 2. \text{ What is the value of } f''(4)?$$

31. Let  $f$  be a function with  $f(3) = 2$ ,  $f'(3) = -1$ ,  $f''(3) = 6$ , and  $f'''(3) = 12$ . Which of the following is the third-degree Taylor polynomial for  $f$  about  $x = 3$ ?

32. Find the interval of convergence of the Maclaurin series for  $f(x) = e^{-2x}$ .

33. Use the 4<sup>th</sup> degree Taylor Series for  $\sin x$  about  $x = 0$  to determine whether  $f$  has a relative minimum, relative maximum, or neither at  $x = 0$ .

34. If  $f(x) = e^{-x^2}$ . Write the first four nonzero terms of the Taylor series for  $\int_0^x e^{-t^2} dt$

about  $x = 0$ ,