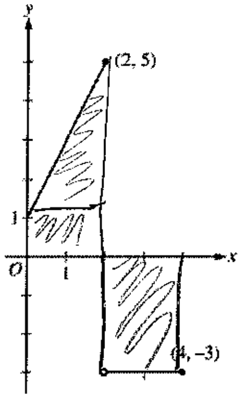
Answer Key *

INTEGRALS AND THEIR APPLICATIONS REVIEW

NO CALCULATOR

1. 

Graph of f

f(x)

The graph of f is shown at the left for $0 \le x \le 4$. What is the value of $\int_0^4 f(x)\,dx$? area

(A) -1 (B) 0 (C) 2 (D) 6 (E) 12

$\frac{1}{2}(2)(4) + (2)(1) - (2)(3)$

$4 + 2 - 6 = 0$

2. Which of the following integrals gives the $\boxed{\text{length}}$ of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1+\frac{1}{x^2}}\,dx$ (B) $\int_1^2 \left(1+\frac{1}{x^2}\right)dx$ (C) $\int_1^2 \sqrt{1+e^{2x}}\,dx$

(D) $\int_1^2 \sqrt{1+(\ln x)^2}\,dx$ (E) $\int_1^2 \left(1+(\ln x)^2\right)dx$

arc length $= \int_a^b \sqrt{1+f'(x)}\,dx$

$= \int_1^2 \sqrt{1+\left(\frac{1}{x}\right)^2}\,dx$

3. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x(x^2-3)^5\,dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5\,du$ (B) $\int_{-2}^{13} u^5\,du$ (C) $\frac{1}{2}\int_{-2}^{13} u^5\,du$

(D) $\int_{-1}^{4} u^5\,du$ (E) $\frac{1}{2}\int_{-1}^{4} u^5\,du$

$u = x^2 - 3$

$du = 2x\,dx$

$u = (-1)^2 - 3 = -2$

$u = (4)^2 - 3 = 13$

$\frac{1}{2}\int_{-2}^{13} u^5\,du$

4.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

start

A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

$50 + (3)(6.2) + (5)(5.9) + 3(5.6)$

$= 50 + 18.6 + 29.5 + 16.8 = 114.9$

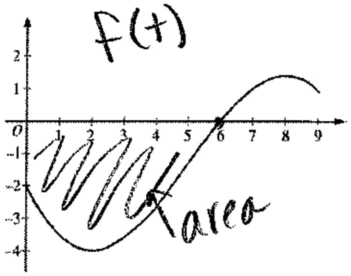
5. $\int_1^4 t^{-3/2}\,dt =$

$\frac{t^{-1/2}}{-1/2} = -2t^{-1/2} = \frac{-2}{\sqrt{t}}\Big]_1^4 = \frac{-2}{\sqrt{4}} - \left(\frac{-2}{\sqrt{1}}\right)$

$= -1 + 2 = 1$

(A) -1 (B) $-\frac{7}{8}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1

6. $f(t)$



Graph of f

The graph of a differentiable function is shown. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$ (B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$ (D) $h''(6) < h(6) < h'(6)$

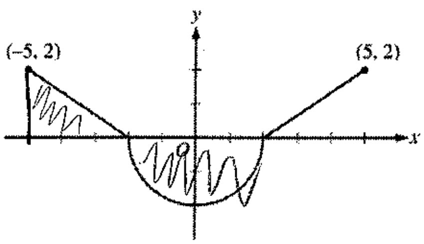
(E) $h''(6) < h'(6) < h(6)$

$h(6) = \int_0^6 f(t)\,dt = \text{area} = \text{neg.}$

$h'(6) = f(6) = 0$

$h''(6) = f'(6) = \text{slope} = \text{pos.}$

7.



Graph of f' $f'(x)$

The graph of f', the derivative of a function f, consisits of two line segments and a seimicircle, as show in the diagram. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$ (B) $2\pi - 3$ (C) $2\pi - 5$

(D) $6 - 2\pi$ (E) $4 - 2\pi$

$\underbrace{\int_{-5}^{2}}_{\text{area}} f'(x) = f(2) - f(-5)$

$\frac{1}{2}(3)(2) - \frac{1}{2}\pi(2)^2 = 1 - f(-5)$

$3 - 2\pi = 1 - f(-5) \rightarrow 2 - 2\pi = f(-5)$

8. $\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is

$(x+2)(x+1)$

(A) $\ln(8)$ (B) $\ln\left(\frac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

$u = \cancel{x^2+3x+8}$

$du = \cancel{2x+3}$

$\frac{5x+8}{(x+2)(x+1)} = \frac{A}{x+2}\,(x+1) + \frac{B}{x+1}\,(x+2)$

$5x+8 = A(x+1) + B(x+2)$

$\underline{x=-1}$: $5(-1)+8 = B(-1+2) \rightarrow 3 = B$

$\underline{x=-2}$: $5(-2)+8 = A(-2+1) \rightarrow -2 = -A \quad A = 2$

$\int \frac{2}{x+2} + \int \frac{3}{x+1}$

$2\ln|x+2| + 3\ln|x+1| \Big]_0^1$

$= 2\ln|1+2| + 3\ln|1+1| - 2\ln|0+2| - 3\ln|0+1|$

$= 2\ln 3 + 3\ln 2 - 2\ln 2 - \cancel{3\ln 1}$

$= \ln 8 + \ln 9 - \ln 4$

$= \ln \frac{72}{4} = \ln 18$

9. $\int 5x\left(\sqrt{x} - x^2\right) dx =$

$\int 5x(x^{1/2} - x^2)\,dx$

$\int 5x^{3/2} - 5x^3\,dx$

$\frac{5x^{5/2}}{5/2} - \frac{5x^4}{4}$

$5 \cdot \frac{2}{5}$

$2x^{5/2} - \frac{5}{4}x^4 + C$

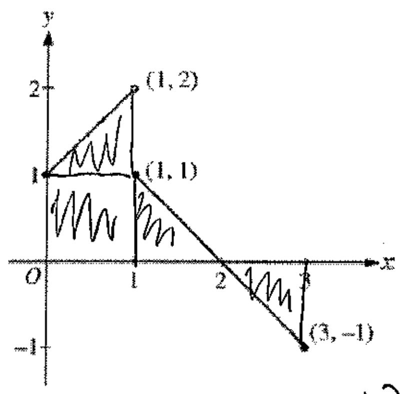
(A) $\frac{15\sqrt{x}}{2} - 15x^2 + C$

(B) $5x - \frac{5x^4}{4} + C$

(C) $2x^{5/2} - \frac{5x^4}{4} + C$

(D) $\frac{25x^{5/2}}{2} - \frac{5x^4}{4} + C$

(E) $\frac{5x^{7/2}}{3} - \frac{5x^6}{6} + C$

10. 

Graph of f

The graph of the function f consists of two line segments. The value of $\int_0^3 |f(x)|\,dx$ is ABSOLUTE VALUE!

(A) $-\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $\frac{3}{2}$

(D) $\frac{5}{2}$ (E) nonexistent

$(1)(1) + (1)(1)(\frac{1}{2}) + (1)(1)(\frac{1}{2}) + (1)(1)(\frac{1}{2})$

$1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2.5$

11. If $\frac{1}{2}\int_0^k \frac{2x}{x^2 + 4}\,dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

(A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln\sqrt{2}\right)$

$u = x^2 + 4$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u}$

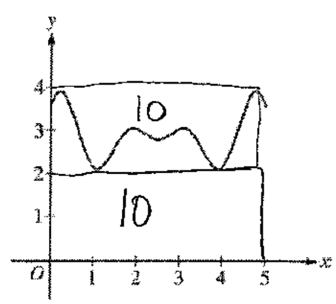
$\frac{1}{2}\ln|x^2+4|\Big]_0^k = \frac{1}{2}\ln|k^2+4| - \frac{1}{2}\ln|4| = \frac{1}{2}\ln\left(\frac{k^2+4}{4}\right) = \frac{1}{2}\ln(4)$

$4 = \frac{k^2+4}{4}$

$16 = k^2 + 4$

$12 = k^2$

$k = \sqrt{12}$

12. 

Graph of f'

f'(x)

The graph of f', the derivative of f, is shown. If $f(0) = 20$, what could be the value of $f(5)$?

(A) 15 (B) 20 (C) 25

(D) 35 (E) 40

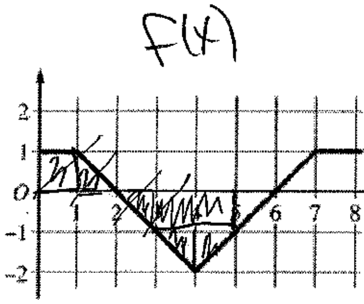
area = Less than 20 but more than 10

$\underbrace{\int_0^5 f'(x)}_{} = f(5) - f(0)$

area $= f(5) - 20$

area $+ 20 = f(5)$

$15 + 20 = 35$ (approx.)

13. 

Graph of f

The graph of the function f in the figure consists of four line segments. Let g be the function defined by $g(x)=\int_0^x f(t)\,dt$.

Which of the following is an equation of the line tangent to the graph of g at $x=5$?

(A) $y+1=x-5$ (B) $y-2=x-5$

(C) $y-2=-1(x-5)$ (D) $y+2=x-5$

(E) $y+2=-1(x-5)$

point: $g(5)=\int_0^5 f(t)\,dt = -2$ area

$(5,-2)$

slope: $g'(5)=f(5)=-1$

$y+2=-1(x-5)$

14. If $\int_1^x f(t)\,dt = \frac{20x}{\sqrt{4x^2+21}} - 4$, then $\int_1^\infty f(t)\,dt$ is

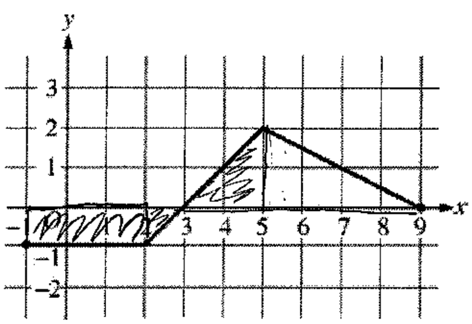
$\lim_{b\to\infty} \int_1^b f(t)\,dt$

(A) 6 (B) 1 (C) −3 (D) −4 (E) divergent

$\lim_{b\to\infty} \frac{20b}{\sqrt{4x^2+21}} - 4 = \lim_{b\to\infty} \frac{\frac{20b}{b}}{\sqrt{\frac{4b^2}{b^2}+\frac{21}{b^2}}} - 4 = \lim_{b\to\infty} \frac{20}{\sqrt{4+\frac{21}{b^2}}} - 4$

CALCULATOR ACTIVE

$= \frac{20}{\sqrt{4}} - 4 = 10-4=6$

15. 

The graph of the piecewise linear function f is shown. What is the value of $\int_{-1}^{9}(3f(x)+2)\,dx$?

$3\underbrace{\int_{-1}^{9} f(x)}_{3\,(\text{area})} + \int_{-1}^{9} 2$

(A) 7.5 (B) 9.5 (C) 27.5

(D) 47 (E) 48.5

$3\left(-3+-\frac{1}{2}+2+4\right)$

$3(2.5) + 2x\Big]_{-1}^{9}$

$2(9)-2(-1)$
$=20$

$20+7.5=$
27.5

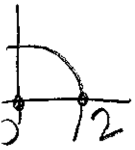
16. What is the average value of $y=\sqrt{\cos x}$ on the interval $0 \le x \le \frac{\pi}{2}$?

(A) −0.637 (B) 0.500 (C) 0.763 (D) 1.198 (E) 1.882

$\frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} \sqrt{\cos x}\,dx = 0.763$

17. Let R be the region in the first quadrant bounded above by the graph of $y=\ln(3-x)$, for $0 \le x \le 2$. R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 0.442 (B) 1.029 (C) 1.296 (D) 3.233 (E) 4.071



$V=\int_a^b [f(x)-g(x)]^2 = \int_0^2 [\ln(3-x)]^2\,dx = 1.029$

18. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $\boxed{a(t) = \frac{t+3}{\sqrt{t^3+1}}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 (E) 11.710

$\int_0^3 \frac{t+3}{\sqrt{t^3+1}}\,dt = V(3) - V(0)$ 5 $\int_0^3 \frac{t+3}{\sqrt{t^3+1}}\,dt + 5 = V(3)$

19. The function h is differentiable, and for all values of x, $\boxed{h(x) = h(2-x)}$. Which of the following statements must be true?

continuous

I. $\int_0^2 h(x)\,dx > 0$

II. $h'(1) = 0$

~~III.~~ $h'(0) = h'(2) = 1$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

$h'(0) = h'(2-0)(-1) = h'(2)(-1)$

$h'(x) = h'(2-x)(-1)$

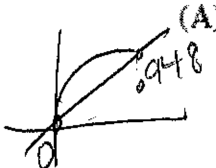
$\underbrace{h'(1)}_{0} = \underbrace{h'(2-1)(-1)}_{0}$

$0 = 0$

only way they will be equal

20. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin(2x)$ and $y = x$

(A) 0.208 (B) 0.210 (C) 0.266 (D) 0.660 (E) 0.835



$\int_0^{.948} \sin(2x) - x\,dx$

21. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

$200 + \int_0^4 -6.89e^{-.053t}\,dt = 175°F$

22.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

Trapezoidal Approx = closest to exact

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154

$\frac{1}{2}(\frac{1}{2})(0+4) + \frac{1}{2}(\frac{1}{2})(4+10) + \frac{1}{2}(\frac{1}{2})(10+18) + \frac{1}{2}(5)(18+28) + (\frac{1}{2})(\frac{1}{2})(28+40) + \frac{1}{2}(\frac{1}{2})(40+54)$

≈ 63.5

23. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

$\pi \int_2^5 (\sqrt{y-2})^2\,dy$

$0 = \sqrt{y-2}$

$0 = y-2$

$y = 2$