Answer Key *

DERIVATIVES AND THEIR APPLICATIONS REVIEW

NO CALCULATOR

1. If $y = \sin^3 x$, then $\frac{dy}{dx} =$ $(\sin x)^3$ $\quad 3(\sin x)^2(\cos x)$

(A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ (E) $3\sin^2 x \cos x$

Implicit Diff.

2. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$ $(y)\frac{1}{\sqrt{1-x^2}} = \frac{1}{y}\frac{dy}{dx}(y) \rightarrow \frac{y}{\sqrt{1-x^2}} = \frac{dy}{dx}$

(A) $\frac{y}{\sqrt{1-x^2}}$ (B) $\frac{xy}{\sqrt{1-x^2}}$ (C) $\frac{y}{1+x^2}$ (D) $e^{\arcsin x}$ (E) $\frac{e^{\arcsin x}}{1+x^2}$

3. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$? $\rightarrow \frac{dy}{dx} = \frac{(x+2)(1)-(x)(1)}{(x+2)^2} = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2} = \frac{1}{2}$

$4 = (x+2)^2$

(A) (0, 0) only (B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ (C) (0, 0) and (-4, 2) (D) (0, 0) and $\left(4, \frac{2}{3}\right)$ (E) none $\pm 2 = x+2$

$\uparrow \frac{0}{0+2} = 0$ $\quad \uparrow \frac{-4}{-4+2} = \frac{-4}{-2} = 2$ $\quad -2 \pm 2 = x$

$x = -4,\ x = 0$

4. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$? $\sin 0 = 0$, $\cos 0 = 1$

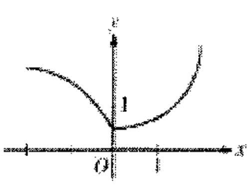
(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

$x = (1+2\sin\theta)(\cos\theta)$ $\quad y = (1+2\sin\theta)(\sin\theta)$

$$\frac{dy}{dx} = \frac{(2\cos\theta)(\sin\theta) + (1+2\sin\theta)(\cos\theta)}{(2\cos\theta)(\cos\theta) + (1+2\sin\theta)(-\sin\theta)} = \frac{(2)(0) + (1+0)(1)}{(2)(1) + (1+0)(0)} = \frac{1}{2}$$

5.

f(x)



Graph of f

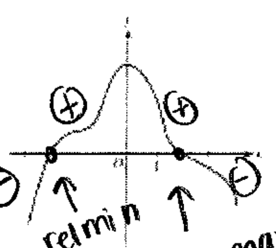
The function f, whose graph is shown at the left, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$. ✓
(B) f is differentiable at $x = 0$. b/c cusp at $x = 0$
(C) f has a critical point at $x = 0$. ✓
(D) f has an absolute minimum at $x = 0$. ✓
(E) The concavity of the graph of f changes at $x = 0$.

Conc. down to concave up

6.

f'(x)



Graph of f'

The graph of f', the derivative of f, is shown at the left. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$
~~II.~~ The graph of f has a point of inflection at $x = -2$. $\rightarrow$ f'(x) doesn't change from decr ↔ incr
III. The graph of f is concave down for $0 < x < 4$.

f'(x) is decr. on $0 < x < 4$ $\rightarrow$ f(x) is concave ↓

7.

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

Concavity

point of inflection at $x=1$

Let f be a function that is twice differentiable on $-2 < x < 2$ and staisfies the conditions in the table. If $f(x) = f(-x)$, what are the x-coordinates f the points of inflection of the graph of f on $-2 < x < 2$?

(A) $x = 0$ only (B) $x = 1$ only (C) $x = 0$ and $x = 1$ (D) $x = -1$ and $x = 1$ (E) none

b/c $f(1) = f(-1)$

8. If $f(x) = \dfrac{x^2 + 3x + 2}{x + 3}$, then $f'(x) =$

Quotient rule

(A) $2x + 3$

(B) $\dfrac{-x^2 - 6x - 7}{(x+3)^2}$

(C) $\dfrac{x^2 + 6x + 7}{(x+3)^2}$

(D) $\dfrac{x^2 + 12x + 11}{(x+3)^2}$

(E) $\dfrac{3x^2 + 12x + 11}{(x+3)^2}$

$f'(x) = \dfrac{(x+3)(2x+3) - (x^2+3x+2)(1)}{(x+3)^2}$

$= \dfrac{2x^2+3x+6x+9-x^2-3x-2}{(x+3)^2}$

$= \dfrac{x^2+6x+7}{(x+3)^2}$

9. Which of the following is the equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point (1, -3)?

Implicit Diff

$2x + (-3)(y) + (-3x)\left(\frac{dy}{dx}\right) = 0$

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

$-3x\frac{dy}{dx} = -2x + 3y$

$\dfrac{dy}{dx} = \dfrac{-2x+3y}{-3x}$

$\dfrac{-2(1)+3(-3)}{-3(1)}$

$= \frac{11}{3}$ slope

10. If $y = \frac{1}{2}x^{4/5} + \dfrac{3}{x^5}$, then $\dfrac{dy}{dx} =$

$-3x^{-5}$

(A) $\dfrac{2}{5x^{1/5}} + \dfrac{15}{x^6}$

(B) $\dfrac{2}{5x^{1/5}} + \dfrac{15}{x^4}$

(C) $\dfrac{2}{5x^{1/5}} - \dfrac{3}{5x^4}$

(D) $\dfrac{2x^{1/5}}{5} + \dfrac{15}{x^6}$

(E) $\dfrac{2x^{1/5}}{5} - \dfrac{3}{5x^4}$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{4}{5}\right)x^{-1/5} + 15x^{-6}$

$= \dfrac{2}{5x^{1/5}} + \dfrac{15}{x^6}$

11. A function f has first derivative given by $f'(x) = x(x-3)^2(x+1)$. For what values of x does f have a relative maximum?

(A) -1 only

(B) 0 only

(C) -1 and 0 only

(D) -1 and 3 only

(E) -1, 0, and 3

$x = 0$ $x = 3$ $x = -1$

critical pts

-2 -.5 1 4

(-)(+)(-) -1 (-)(+)(+) 0 (+)(+)(+) 3 (+)(+)(+)

pos neg pos. pos.

rel. max

+ −

12. If $\displaystyle\lim_{h \to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a?

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\dfrac{1}{2}$ (E) 2

Limit Def. of Derivative

$\frac{d}{dx}\arcsin(a) = 2$

$\dfrac{1}{\sqrt{1-x^2}} = 2$

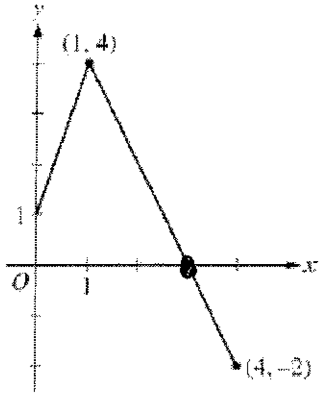
$1 = 2\sqrt{1-x^2}$

$\frac{1}{2} = \sqrt{1-x^2}$

$\frac{1}{4} = 1 - x^2$

$-\frac{3}{4} = -x^2$

$\frac{\sqrt{3}}{2} = x$

13. 

Graph of f

$f(x)$

The graph of the function f, consisting of two line segments, is shown in the figure at the left. Let g be the function given by $g(x) = 2x + 1$, and let h be the function given by $h(x) = f(g(x))$. What is the value of $h'(1)$?

(A) -4 (B) -2 (C) 4

(D) 6 (E) nonexistent

$g(1) = 2(1)+1$

$g(1) = 3$

$g'(x) = 2$

Chain rule $\rightarrow$ $h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(3) \cdot (2)$

slope

$\left(\frac{-2-4}{4-1}\right)(2) = \left(\frac{-6}{3}\right)(2) = -4$

14. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2, 1)$?

(A) $\frac{dy}{dx}$ exists at $(2, 1)$, but there is no tangent line at that point.

(B) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is horizontal.

(C) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is neither horizontal nor verti

(D) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is vertical.

(E) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is horizontal.

$2x + (-1)(y) + (-x)\left(\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-x+2y) = -2x+y$

$\frac{dy}{dx} = \frac{-2x+y}{-x+2y}$

$= \frac{-2(2)+1}{-2+2(1)}$

$= \frac{-3}{0}$ undef.

$\downarrow$

vert. tangent

15. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

check critical pts & endpts

$g'(x) = 12x^2 + 6x - 6$

$6(2x^2 + x - 1)$

$6(2x-1)(x+1) = 0$ $x = 1/2, -1$

x	$g(x)$
-2	$4(-2)^3+3(-2)^2-6(-2)+1$ -7
1	2
$1/2$	$-3/4$
-1	6

16. If $x = t^2 - 1$ and $y = \ln t$, what is $\frac{d^2y}{dx^2}$ in terms of t?

(A) $-\frac{1}{2t^4}$ (B) $\frac{1}{2t^4}$ (C) $-\frac{1}{t^3}$ (D) $-\frac{1}{2t^2}$ (E) $\frac{1}{2t^2}$

$\frac{dy}{dx} = \frac{\frac{1}{t}}{2t} = \frac{1}{2t^2} \rightarrow \frac{d^2y}{dx^2} = \frac{-t^{-3}}{2t} = \frac{-1}{2t^4}$

$= \frac{1}{2}t^{-2}$

17.

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

Continuous

The function f has the properties indicated in the table above. Which of the following must be true?

~~(A)~~ f is continuous at $x = -1$

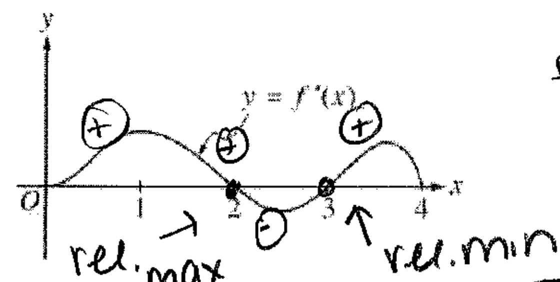
~~(B)~~ f is continuous at $x = 0$

(C) f is continuous at $x = 1$ b/c $\lim\limits_{x\to a^-} = \lim\limits_{x\to a^+} = f(a) = 2$

(D) f is differentiable at $x = 0$

(E) f is differentiable at $x = 1$.

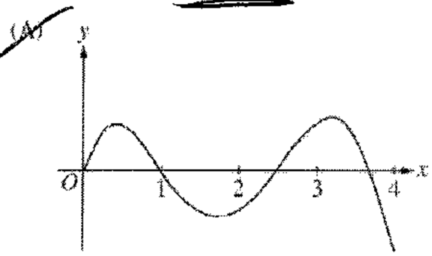
18.



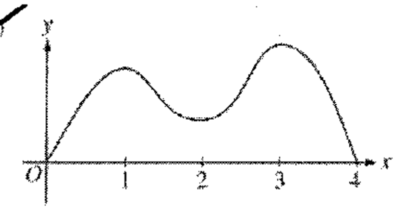
f'(x)

The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

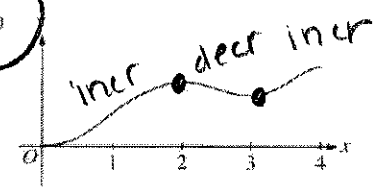
~~(A)~~



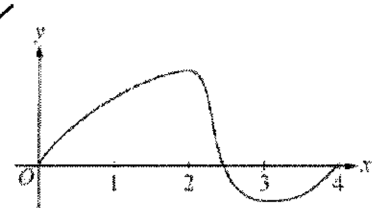
~~(B)~~



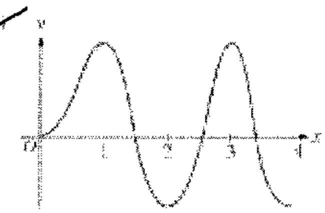
(C)



~~(D)~~



~~(E)~~

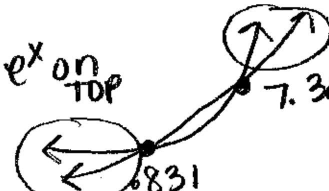


Window
Ymax = 2000

$Y_1 = e^x$
$Y_2 = 4x^3$

CALCULATOR ACTIVE

e^x on TOP



19. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = x^4$. On what intervals is the $\boxed{\text{rate of change}}$ of $f(x)$ greater than the rate of change of $g(x)$?

derivative

(A) (0.831, 7.384) only (B) $(-\infty, 0.831)$ and $(7.384, \infty)$ (C) $(-\infty, \infty)$

(D) (-0.816, 1.430) and $(8.613\ \infty)$ (E) $(-\infty, 0.816)$ and (1.430, 8.613)

20. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{(x^4 - 2x^2 + 1)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

Graph

f(x) is conc ↓ when f'(x) is decr.

(A) (-0.418, 0.418) only (B) (-1, 1) (C) (-1.354, -0.409)
(D) (-1.5, -1) and (0, 1) (E) (-1.5, -1.354), (-0.409, 0) and (1.354, 1.5)

f'(x)

$(-1.5, -1) \cup (0, 1)$

21. The fuel consumption of a car, in miles per gallon (mpg), is modeled by $F(t) = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)}$, where s is the speed of the car, in miles per hour. If the car is traveling at 50 miles per hour and its speed is changing at the rate of 20 miles/hour^2, what is the rate at which its fuel consumption is changing?

$S = 50$

$\frac{ds}{dt} = 20$

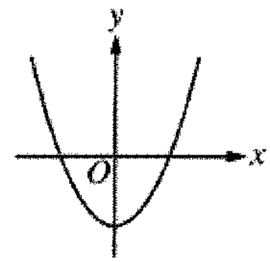
(A) 0.215 mpg per hour (B) 4.299 mpg per hour (C) 19.793 mpg per hour
(D) 25.793 mpg per hour (E) 515.855 mpg per hour

$\frac{dF}{dS} = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)}\left(\frac{1}{20} - \frac{s}{1200}\right)\frac{ds}{dt} = 6e^{\left(\frac{50}{20} - \frac{50^2}{2400}\right)}\left(\frac{1}{20} - \frac{50}{1200}\right)(20) = 4.299$

22. The derivative of a function f is increasing for $x < 0$ and decreasing for $x > 0$. Which of the following could be the graph of f?

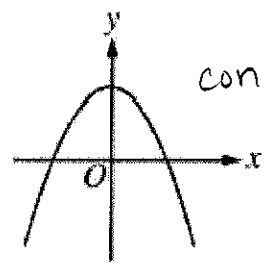
f'(x) incr → conc ↑ f'(x) decr → conc ↓

~~(A)~~

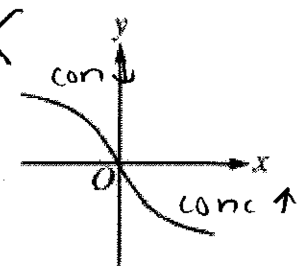


con ↑

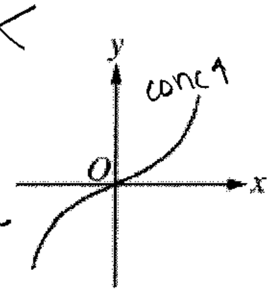
~~(B)~~



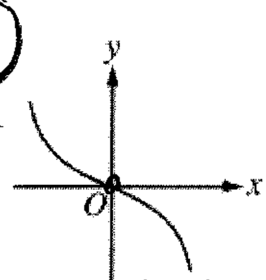
~~(C)~~



~~(D)~~



(E)



23. Let f be a function whose derivative is given by $f'(x) = \ln(x^4 + 5x^3 + x^2 - 7x + 28)$. On the open interval $(-4, 1)$, at which of the following values of x does f attain a relative maximum?

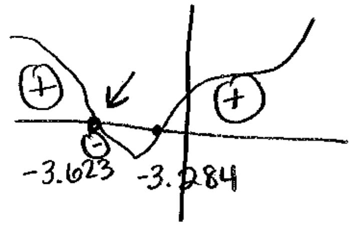
(A) -3.623 only

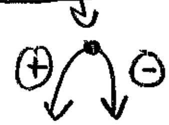
(B) -0.871 only

(C) -3.623 and -3.284

(D) -3.459 and 0.581 only

(E) -3.459, -0.871, and 0.581





f(x) is incr.   f'(x) is decr

24. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
−1	4
0	3
1	1

f(x) is decr

(B)

x	$f(x)$
−1	4
0	4
1	4

(C)

x	$f(x)$
−1	4
0	5
1	6

1, 1 constant

(D)

x	$f(x)$
−1	4
0	5
1	7

1, 2 incr

(E)

x	$f(x)$
−1	4
0	6
1	7

2, 1 f'(x) decr

25. The volume of a certain cone for which the sum of its radius, r, and height is constant is given by $V = \frac{1}{3}\pi r^2(10 - r)$. The rate of change of the radius of the cone with respect to time is 6. In terms of r, what is the rate of change of the volume of the cone with respect to time?

$\frac{dV}{dt} = (\frac{2}{3}\pi r)(10-r)(\frac{dr}{dt}) + (\frac{1}{3}\pi r^2)(-1)(\frac{dr}{dt})$

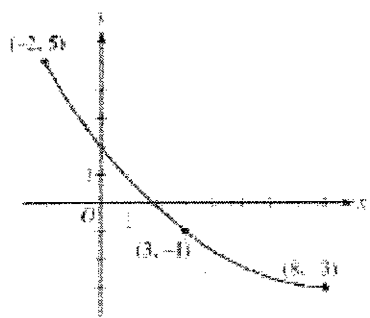
$\frac{dr}{dt} = 6$

(A) $-24\pi r$ (B) $6\pi r$ (C) $\frac{20}{3}\pi r - \pi r^2$ (D) $16\pi r - \frac{4}{3}\pi r^2$ (E) $40\pi r - 6\pi r^2$

$4\pi r(10-r) - 2\pi r^2$

$40\pi r - 4\pi r^2 - 2\pi r^2$

26.



Graph of f

f(x)

A portion of the graph of a differentiable function f is shown at the left. If the value $c = 3$ satisfies the conclusion of the Mean Value Theorem applied to f on the open interval $-2 < x < 8$, what is the slope of the line tangent to the graph at $x = 3$?

(A) $-\frac{7}{5}$ (B) $-\frac{5}{4}$ (C) $-\frac{4}{5}$

(D) $-\frac{5}{7}$ (E) $-\frac{1}{5}$

$f'(c) = \frac{f(b) - f(a)}{a - b}$

$f'(3) = \frac{f(8) - f(-2)}{8 - (-2)}$

slope $\to f'(3) = \frac{-3-5}{10} = -\frac{4}{5}$

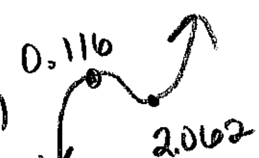
27. Let f be a function with derivative given by $f'(x) = x^3 - 5x^2 + e^x$. On which of the following intervals is the graph of f concave down?

Graph

(A) $(-\infty, 0.117)$ only (B) $(-\infty, 1.144)$ (C) $(0.116, 2.062)$

(D) $(0.673, 2.863)$ (E) $(2.863, \infty)$

f'(x)



f'(x) is decr $\to$ f(x) concave ↓