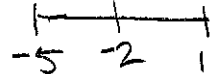


AP Calculus BC - Midterm Review Warm-up #8

Name: Answer Key *

1) Determine the interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n} \quad C = -2$$



$$\lim_{n \rightarrow \infty} \left| \frac{(x+2)^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{(x+2)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x+2}{3} \cdot \frac{n}{n+1} \right| = \left| \frac{x+2}{3} \right| \lim_{n \rightarrow \infty} \left| \frac{n}{n+1} \right| = \left| \frac{x+2}{3} \right|$$

$$\left| \frac{x+2}{3} \right| < 1 \quad |x+2| < 3 \quad R=3$$

x=1: $\sum \frac{(3)^n}{n3^n} = \sum \frac{1}{n}$ p-series diverges

x=-5: $\sum \frac{(-3)^n}{n3^n} = \sum \frac{(-1)^n}{n}$ alt series $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$ $\frac{1}{n+1} \leq \frac{1}{n}$ converges

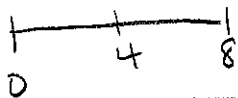
[-5, 1)

2) Find the interval of convergence of f'(x) if

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n4^n} \quad C = 4$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+2}(x-4)^{n+1}}{(n+1)4^{n+1}} \cdot \frac{n4^n}{(-1)^{n+1}(x-4)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x-4}{4} \right| = \left| \frac{x-4}{4} \right| < 1$$

$$|x-4| < 4 \quad R=4$$



$f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^{n-1}}{4n}$

x=0: $\sum \frac{(-1)^{n+1}(-4)^{n-1}}{4n} = \sum \frac{(-1)^n(-1)^1(-4)^n(-4)^{-1}}{4n} = \sum \frac{(-1)^n(-1)(-1)^n(-4)^{-1}}{4n}$

$$= \sum \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \dots \text{ diverges}$$

x=8: $\sum \frac{(-1)^{n+1}(4)^{n-1}}{4n} = \sum \frac{(-1)^n(-1)^1(4)^n(4)^{-1}}{4n} = \sum \frac{(-1)^n}{4}$ alt series test

(0, 8)

$\lim_{n \rightarrow \infty} \frac{1}{4} \neq 0$ diverges by nth term