AP Calculus BC - Midterm Review Warm-up #8

Name: Answer Key *

1) Determine the interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n}$$

$C=-2$

$-5 \quad -2 \quad 1$

$\lim_{n\to\infty}\left|\frac{(x+2)^{n+1}}{(n+1)3^{n+1}}\cdot\frac{n\cancel{3^n}}{\cancel{(x+2)^n}}\right| = \lim_{n\to\infty}\left|\frac{x+2}{3}\frac{(n)}{(n+1)}\right| = \left|\frac{x+2}{3}\right|\lim_{n\to\infty}\left|\frac{n}{n+1}\right| = \left|\frac{x+2}{3}\right|$

$\left|\frac{x+2}{3}\right| < 1 \quad |x+2| < 3 \quad R=3$

$\underline{x=1}$: $\sum_1 \frac{\cancel{(3^n)}}{n\cancel{3^n}} \quad \sum_1 \frac{1}{n}$ p-series diverges

$\underline{x=-5}$ $\sum_1 \frac{(-3)^n}{n3^n} = \sum_1 \frac{(-1)^n}{n}$ alt series

$\lim_{n\to\infty}\frac{1}{n}=0 \quad \frac{1}{n+1}\le\frac{1}{n}$

converges

$\boxed{[-5,1)}$

2) Find the interval of convergence of f'(x) if

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n4^n}$$

$C=4$

$\lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n+2}}(x-4)^{n+1}}{(n+1)4^{n+1}}\cdot\frac{n\cancel{4^n}}{\cancel{(-1)^{n+1}}\cancel{(x-4)^n}}\right| = \lim_{n\to\infty}\left|\frac{x-4}{}\frac{(n)}{(n+1)(4)}\right| = \left|\frac{x-4}{4}\right| < 1$

$|x-4| < 4 \quad R=4$

$0 \quad 4 \quad 8$

$f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^{n-1}}{4n}$

$\underline{x=0}$: $\sum_1 \frac{(-1)^{n+1}(-4)^{n-1}}{4n} = \sum_1 \frac{(-1)^n(-1)^1(-4)^n(-4)^{-1}}{4n} = \sum_1 (-1)^n(-1)(-1)^n(-4)^{-1}$

$= \sum_1 \frac{1}{4} = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\ldots$ diverges

$\underline{x=8}$: $\sum_1 \frac{(-1)^{n+1}(4)^{n-1}}{4n} = \sum_1 \frac{(-1)^n(-1)^1\cancel{(4)^n}(4)^{-1}}{\cancel{4n}} = \sum_1 -\frac{(-1)^n}{4}$ alt series test

$\lim_{n\to\infty}\frac{1}{4}\neq 0$ diverges by nth term

$\boxed{(0,8)}$