

AP Calculus BC - Midterm Review Warm-up #6

Name: Answer Key*

1) Find the McLaurin polynomial of degree 3 for $f(x) = 2e^x$.

$c=0$

$f(x) = 2e^x \rightarrow 2e^0 = 2$

$f'(x) = 2e^x \rightarrow 2$

$f''(x) = 2e^x \rightarrow 2$

$f'''(x) = 2e^x \rightarrow 2$

$P_3(x) = 2 + 2x + \frac{2x^2}{2!} + \frac{2x^3}{3!}$

$P_3(x) = 2 + 2x + x^2 + \frac{1}{3}x^3$

2) Use Taylor's Theorem to obtain an upper bound for the error of the approximation.

$\cos(0.9) \approx 1 - \frac{(0.9)^2}{2!} + \frac{(0.9)^4}{4!} \quad n=4$

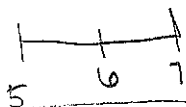
$R_4(x) = \frac{f^{(5)}(x)}{5!} (x-c)^5 = \frac{1}{5!} (0.9)^5 = 0.005$

3) Find the interval of convergence of $f'(x)$ if

$\frac{1}{n} (x-6)^n \quad f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-6)^n}{n} \quad c=6$

$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+2} (x-6)^{n+1}}{(n+1)} \cdot \frac{n}{(-1)^{n+1} (x-6)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x-6)(n)}{n+1} \right| = |x-6| \lim_{n \rightarrow \infty} \left| \frac{n}{n+1} \right| = |x-6|$

$|x-6| < 1 \quad R=1$



$f'(x) = \sum_{n=1}^{\infty} (-1)^{n+1} (x-6)^{n-1}$

$x=5: \sum_{n=1}^{\infty} (-1)^{n+1} (-1)^{n-1} = \sum_{n=1}^{\infty} (-1)^n (-1)^{-1} (-1)^n (-1)^{-1} = \sum_{n=1}^{\infty} -1 = -1 -1 -1 \dots$
diverges

$x=7: \sum_{n=1}^{\infty} (-1)^{n+1} (1)^{n-1} = \sum_{n=1}^{\infty} (-1)^n (-1)^{-1} (1)^n (1)^{-1} = \sum_{n=1}^{\infty} (-1)^n (-1)$
Alt Series Test
 $\lim_{n \rightarrow \infty} 1 = 1 \neq 0$ diverges by nth term test

$(5, 7)$