AP Calculus BC - Midterm Review Warm-up #6

Name: Answer Key*

1) Find the McLaurin polynomial of degree 3 for $f(x) = 2e^x$.

$c=0$

$f(x) = 2e^x \rightarrow 2e^0 = 2$

$f'(x) = 2e^x \rightarrow 2$

$f''(x) = 2e^x \rightarrow 2$

$f'''(x) = 2e^x \rightarrow 2$

$P_3(x) = 2 + 2x + \frac{2x^2}{2!} + \frac{2x^3}{3!}$

$\boxed{P_3(x) = 2 + 2x + x^2 + \frac{1}{3}x^3}$

2) Use Taylor's Theorem to obtain an upper bound for the error of the approximation.

$$\cos(0.9) \approx 1 - \frac{(0.9)^2}{2!} + \frac{(0.9)^4}{4!} \qquad n=4$$

$R_4(x) = \frac{f^5(x)}{5!}(x-c)^5 = \frac{1}{5!}(0.9)^5 = \boxed{0.005}$

3) Find the interval of convergence of $f'(x)$ if

$\frac{1}{n}(x-6)^n$

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-6)^n}{n}$$

$c=6$

$\lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n+2}}(x-6)^{n+1}}{(n+1)} \cdot \frac{(n)}{\cancel{(-1)^{n+1}}\cancel{(x-6)^n}}\right| = \lim_{n\to\infty}\left|\frac{(x-6)(n)}{n+1}\right| = |x-6| \lim_{n\to\infty}\left|\frac{n}{n+1}\right| = |x-6|$

$|x-6| < 1 \qquad R=1$

5 — 6 — 7

$f'(x) = \sum_{n=1}^{\infty} (-1)^{n+1}(x-6)^{n-1}$

$\underline{x=5}$: $\sum_{n=1}^{\infty} (-1)^{n+1}(-1)^{n-1} = \sum (-1)^n(-1)^1(-1)^n(-1)^{-1}$

$= \sum -1 = -1 + -1 + -1 + \ldots$ diverges

$\underline{x=7}$: $\sum_{n=1}^{\infty} (-1)^{n+1}(1)^{n-1} = \sum (-1)^n(-1)^1(1)^n(1)^{-1}$

$= \sum (-1)^n(-1)$ Alt Series Test

$\lim_{n\to\infty} 1 = 1 \neq 0$ diverges by nth term test

$\boxed{(5,7)}$