AP Calculus BC – Midterm Review Warm-up #5

Name: Answer Key*

1) Evaluate the integral:

$\int x\sec x\tan x\,dx$

$u = x \quad v = \sec x$

$du = dx \quad dv = \sec x\tan x\,dx$

$(x)(\sec x) - \int (\sec x)\,dx$

$x\sec x - \ln|\sec x + \tan x| + C$

2) Evaluate:

$\int \frac{5x+1}{x^2+9}\,dx$

$\frac{1}{2}5\int \frac{2x}{x^2+9}\,dx + \int \frac{1}{x^2+9}\,dx$

$u = x^2+9$

$du = 2x\,dx$

$\frac{5}{2}\int \frac{du}{u}$

$\frac{5}{2}\ln|x^2+9|$

$u = x$

$du = dx$

$a = 3$

$\int \frac{du}{u^2+a^2}$

$\frac{1}{3}\arctan\left(\frac{x}{3}\right)$

$\frac{5}{2}\ln|x^2+9| + \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$

3) Determine if the series converges or diverges:

Root Test

$\sum_{n=1}^{\infty}\left(\frac{5n^2-n+6}{2n^2+6n-3}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{5n^2-n+6}{2n^2+6n-3}\right)^n} = \lim_{n\to\infty}\left(\frac{5n^2-n+6}{2n^2+6n-3}\right) = \frac{5}{2} > 1$

diverges

4) Determine if the series converges or diverges:

$\sum_{n=0}^{\infty}\frac{1}{4^n+1}$

$a_n = \frac{1}{4^n+1} \quad b_n = \frac{1}{4^n}$ bigger

$\sum_{n=1}^{\infty}\frac{1}{4^n} = \sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$ Geometric series Test

$r = 1/4$

converges by direct comparison test