AP Calculus BC - Midterm Review Warm-up #4

Name: Answer Key *

1) Evaluate the integral:

$$\frac{1}{5}\int \frac{5\,dx}{\sqrt{16-25x^2}}$$

$u = 5x$
$du = 5\,dx$
$a = 4$

$$\frac{1}{5}\int \frac{du}{\sqrt{a^2-u^2}} = \boxed{\frac{1}{5}\arcsin\left(\frac{5x}{4}\right)+C}$$

2) Evaluate:

$$-\int -\tan x \ln(\cos x)\,dx$$

$u = \ln(\cos x)$
$du = \frac{-\sin x}{\cos x}\,dx = -\tan x\,dx$

$$-\int u\,du$$

$$-\left[\frac{1}{2}u^2\right]+C \qquad \boxed{-\frac{1}{2}\left[\ln(\cos x)\right]^2+C}$$

3) Find the horizontal asymptotes of

$$\frac{dy}{dx} = 4y\left(\frac{8}{4}-\frac{4y}{4}\right) \qquad \frac{dy}{dx} = 4y(2-y)$$

$$\boxed{\text{H.A. } y=0 \text{ \& } y=2}$$

4) Find what the series converges to.

Telescoping Series

$$\frac{1}{n(n+2)} = \frac{A(n+2)}{n(n+2)} + \frac{B(n)}{(n+2)(n)}$$

$1 = A(n+2)+B(n)$

$n=-2$; $1 = B(-2)$, $B = -1/2$

$n=0$: $1 = A(0+2)$, $A = 1/2$

$$\sum_{n=1}^{\infty}\frac{1}{n(n+2)} = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}-\frac{1}{n+2}$$

$$\frac{1}{2}\left[\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{3}}-\frac{1}{5}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)+\left(\cancel{\frac{1}{6}}-\cancel{\frac{1}{8}}\right)+\ldots+\left(\cancel{\frac{1}{n}}-\frac{1}{n+2}\right)\right]$$

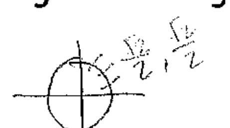
$$\lim_{n\to\infty}\frac{1}{2}\left[1+\frac{1}{2}-\frac{1}{n+2}\right] = \frac{1}{2}\left[1+\frac{1}{2}\right] = \frac{1}{2}\left[\frac{3}{2}\right] = \frac{3}{4}$$

5) Use the Integral Test to determine if the series converges or diverges.

$$\sum_{n=1}^{\infty}\frac{1}{n^2+1}$$

$$\lim_{n\to\infty}\int_1^b \frac{1}{x^2+1}$$

$u = x$
$du = dx$
$a = 1$



$$\int\frac{du}{u^2+a^2} \rightarrow \arctan x\Big]_1^b = \arctan b - \arctan 1 = \arctan b - \frac{\pi}{4}$$

$$\lim_{n\to\infty}\left[\arctan b-\frac{\pi}{4}\right] = \frac{\pi}{2}-\frac{\pi}{4} = \frac{\pi}{4}$$

converges