AP Calculus BC - Midterm Review Warm-up #2

Name: Answer Key*

1) Evaluate the integral:

$$\int e^x sinx dx$$

$u = \sin x \quad v = e^x$

$du = \cos x\,dx \quad dv = e^x dx$

$(\sin x)(e^x) - \int e^x(\cos x)dx$

$u = \cos x \quad v = e^x$

$du = -\sin x\,dx \quad dv = e^x dx$

$-[(\cos x)(e^x) - \int e^x(-\sin x\,dx)]$

$-e^x\cos x + \int e^x(-\sin x)dx$

$-\int e^x \sin x\,dx$

$\int e^x\sin x dx = e^x\sin x - e^x\cos x - \int e^x \sin x dx$

$2\int e^x \sin x dx = e^x \sin x - e^x\cos x$

$\boxed{\int e^x \sin x dx = \frac{1}{2}(e^x\sin x - e^x\cos x)}$

2) Evaluate:

$$\int x\sqrt{x+1}\,dx$$

$u = x+1$

$du = dx$

$u-1 = x$

$\int (u-1)u^{1/2}du$

$\int u^{3/2} - u^{1/2}du$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$

3) Find the number of terms necessary to approximate the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

$a_n = \frac{1}{n} \quad a_{n+1} = \frac{1}{n+1}$

with an error of less than or equal to 0.0001.

$\frac{1}{n+1} \leq .0001$

$\frac{1}{n+1} \leq \frac{1}{10000}$

$n+1 = 10000$

$\boxed{n = 9999} \to \boxed{9999 \text{ terms}}$

4) Determine if the series converges or diverges. If it converges, find the sum.

$$\sum_{n=2}^{\infty} \frac{6}{(n+1)(n+2)}$$

Telescoping Series

$\frac{6}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$

$6 = A(n+2) + B(n+1)$

$n=-2$: $6 = B(-2+1)$

$6 = -B$

$n=-1$: $6 = A(-1+2)$

$6 = A$

$\sum_{n=2}^{\infty} \frac{6}{n+1} - \frac{6}{n+2} = 6\sum_{n=2}^{\infty} \frac{1}{n+1} - \frac{1}{n+2}$

$6[(\frac{1}{3} - \cancel{\frac{1}{4}}) + (\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}) + (\cancel{\frac{1}{5}} - \cancel{\frac{1}{6}})$

$+ (\cancel{\frac{1}{6}} - \cancel{\frac{1}{7}}) + (\cancel{\frac{1}{7}} - \frac{1}{8}) + \ldots (\cancel{\frac{1}{n+1}} - \frac{1}{n+2})]$

$\lim_{n\to\infty} 6[\frac{1}{3} - \frac{1}{n+2}] = 6(\frac{1}{3}) = \boxed{2}$ $\boxed{\text{converges}}$