AP Calculus BC – Midterm Review #1 Name: Answer Key*

1) Evaluate the integral:

$$x^2+10x+\left(\frac{10}{2}\right)^2+29-\left(\frac{10}{2}\right)^2$$

$$(x+5)^2+4$$

$$\int \frac{7}{x^2+10x+29}dx = 7\int \frac{1}{(x+5)^2+4}dx$$

$u=x+5$
$du=dx$
$a=2$

$$7\int \frac{du}{u^2+a^2} = 7\left(\tfrac{1}{2}\right)\arctan\frac{(x+5)}{2}+C$$

$$\boxed{\frac{7}{2}\arctan\frac{x+5}{2}+C}$$

2) Which of the following integrals are divergent?

~~I.~~ $\int_2^\infty \frac{x}{(1+x^2)^2}dx$ II. $\int_1^\infty \cos 2x\,dx$ ~~III.~~ $\int_1^\infty \frac{1}{x^3}dx$

Short-cut Rule: $p=3$, $p>1$, converges

$$\lim_{b\to\infty}\frac{1}{2}\int_2^b \frac{2x}{(1+x^2)^2}dx \quad u=1+x^2,\ du=2x\,dx$$

$$\frac{1}{2}\int\frac{du}{u^2}=\frac{1}{2}\int u^{-2}du=\frac{1}{2}\left[-1u^{-1}\right]$$

$$\left.-\frac{1}{2(1+x^2)}\right]_2^b = \frac{-1}{2(1+b^2)}+\frac{1}{2(1+2^2)} \to \lim_{b\to\infty}\left(\frac{-1}{2(1+b^2)}+\frac{1}{10}\right) = -\frac{1}{\infty}+\frac{1}{10}=\frac{1}{10}$$

$$\lim_{b\to\infty}\int_1^b \cos 2x\,dx$$

$$\left.\frac{1}{2}\sin 2x\right]_1^b$$

$$\lim_{b\to\infty}\left(\tfrac{1}{2}\sin 2b-\tfrac{1}{2}\sin 2\right) = \text{DNE}$$

3) Find the value of the series:

$$\sum_{n=1}^{\infty} 2\left(-\frac{1}{2}\right)^n$$

If $n\neq 0$, must find a.

Geometric Series
Converges b/c $r=-\frac{1}{2}$

$$S=\frac{a}{1-r} \qquad a=2\left(-\tfrac{1}{2}\right)^1=2\left(-\tfrac{1}{2}\right)=-1$$

$$S=\frac{-1}{1-\left(-\frac{1}{2}\right)}=\frac{-1}{1+\frac{1}{2}}=\frac{-1}{\frac{3}{2}}=\boxed{-\frac{2}{3}}$$

4) Determine if the series is convergent or divergent. If the series converges, then determine if it is absolutely convergent or conditionally convergent.

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$$

Alternating Series Test $a_n=\frac{1}{n^{1/2}}$

① $\lim_{n\to\infty}\frac{1}{n^{1/2}}=\frac{1}{\infty}=0$ ✓

② $\frac{1}{\sqrt{n+1}}\le\frac{1}{\sqrt{n}}$ ✓

$\therefore \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ Converges by

$\sum_{n=1}^{\infty}\frac{1}{n^{1/2}} \to$ p-series test, $p=1/2$, diverges

$$\boxed{\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}} \text{ is conditionally convergent}}$$