AP Calculus AB
Final Review

BINGO
(Units 5 - 7)

At t = 0 water begins leaking from a tank at the rate of $L(t) = 5e^{-\frac{(t-3)^2}{2}}$ gal per min, where t is measured in min. How much water ~~has~~ gal leaked out of the tank after 5 minutes? (calculator)

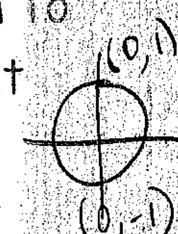
$$\int_0^5 5e^{-\frac{(t-3)^2}{2}}\,dt = \boxed{12.231}$$

$$\int (2x - 3x^2)\ dx$$

$$\frac{2x^2}{2} - \frac{3x^3}{3} + C$$

$$\boxed{x^2 - x^3 + C}$$

Apply Rolle's Theorem to find all values of c that satisfy the theorem.




f(x) = sinx on the interval [0, 2π]

- continuous ✓
- diff ✓
- $f(0) = \sin 0 = 0$ ✓
$f(2\pi) = \sin 2\pi = 0$

$f'(x) = \cos x$

$\cos(c) = 0$

$\boxed{c = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}}$

A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

$v(t) = e^t$

$\int_0^2 v(t)\,dt = \int_0^2 e^t\,dt$

$= e^t \Big]_0^2$

$= e^2 - e^0 = \boxed{e^2 - 1}$

A particle moves along the x-axis and is given by $p(t) = e^{2t} - 5t$. On what interval(s) of t is the particle moving to the left? (Calculator)

$p(t) = e^{2t} - 5t$

$p'(t) = v(t) = 2e^{2t} - 5$

$2e^{2t} - 5 = 0$

$e^{2t} = \frac{5}{2}$

$\ln(\frac{5}{2}) = 2t$

$t = 0.458$

$\longleftarrow 0 \quad | \quad 1 \longrightarrow$

$2e^{2(0)} - 5 \quad 0.458 \quad 2e^{2(1)} - 5$

$\ominus \qquad \oplus$

Left

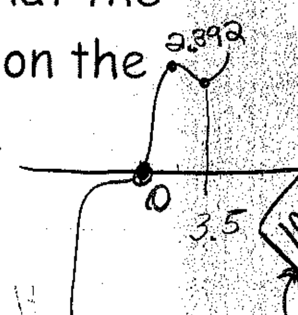
$\boxed{[0, 0.458)}$

$\lim_{x \to 2} \frac{3x^2 - 4x - 4}{x^2 - 4}$

$\lim_{x \to 2} \frac{6x - 4}{2x}$

$\frac{6(2) - 4}{2(2)} = \frac{8}{4} = \boxed{2}$

A particle's position is given by the function $p(t) = 3t + 4.1\sin(t)$. Find the total distance that the particle travels on the interval [0, 3.5]. (Calculator)




$\boxed{10.877}$

$|p(0) - p(2.392)| + |p(2.392) - p(3.5)|$

$|0 - 9.969| + |9.969 - 9.062|$

If $f(x) = x^3 - 3x^2$, what is the absolute minimum on the interval [-1, 3]?

Extreme value Thm

endpts $\underline{x=-1}$ $(-1)^3-3(-1)^2 = -4$

$\underline{x=3}$ $(3)^3-3(3)^2 = 0$

critical values $f'(x) = 3x^2-6x$

$3x^2-6x=0$

$3x(x-2)$

$x=0$ $x=2$

$\underline{x=0}$ $(0)^3-3(0)^2=0$

$\underline{x=2}$ $(2)^3-3(2)^2=-4$

$\boxed{(-1,-4) \text{ \& } (2,-4)}$

Find $f'(x)$.

$$f(x) = \int_{-2}^{x^4} 3\sqrt{t}\,dt$$

$f'(x) = \int_{-2}^{x^4} 3t^{1/2}\,dt$

$3(x^4)^{1/2}(4x^3)$

$3x^2(4x^3) = \boxed{12x^5}$

Find the x-value of the point on the graph of $f(x) = \sqrt{-x+8}$ so that the point (2, 0) is closest to the graph.

$\boxed{x=2.5}$

minimize distance

$d = \sqrt{(x-2)^2+(y-0)^2}$

$= \sqrt{(x-2)^2+(\sqrt{-x+8})^2}$

$= \sqrt{x^2-4x+4-x+8}$

$= \sqrt{x^2-5x+12}$

$= (x^2-5x+12)^{1/2} \rightarrow$ $d'(x) = \frac{1}{2}(x^2-5x+12)^{-1/2}(2x-5)$

$= \dfrac{2x-5}{2\sqrt{x^2-5x+12}}$ $x = 5/2$

$d'$: $-$ | $5/2$ | $+$ $3$

The velocity of a particle is given by $v(t) = (t-2)^2\cos 2t$. What is the average acceleration between $t = 1$ and $t = 3.5$ seconds? (Calculator)

$\dfrac{v(3.5)-v(1)}{3.5-1}$

$= \dfrac{1.696-(-.416)}{2.5}$

$= \boxed{0.845}$

If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $0 \le x \le 2$, then c = ?

$f'(x) = 3x^2 - 4x$ $\quad \frac{f(0)-f(2)}{0-2} = \frac{0-0}{-2}$

$= 0$

$3c^2 - 4c = 0$

$c(3c-4) = 0$

$\cancel{c=0}$ $\boxed{c = 4/3}$

(calculator)

The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume?

$V = \frac{\sqrt{3}}{4} \int_0^2 [2x - x^2]^2 dx$

$= \boxed{0.462}$

Find the particular solution, y = f(x) when $dy/dx = 6x^2 + 6x + 2$ and $f(-1) = 2$.

$\frac{dy}{dx} = 6x^2 + 6x + 2$

$\int dy = \int 6x^2 + 6x + 2 dx$

$y = 2x^3 + 3x^2 + 2x + C$

$(-1, 2)$

$2 = 2(-1)^3 + 3(-1)^2 + 2(-1) + C$

$2 = -2 + 3 - 2 + C$

$2 = -1 + C$

$C = 3$

$\boxed{2x^3 + 3x^2 + 2x + 3}$

| x | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| f(x) | 4 | k | 8 | 12 |

The trapezoidal approximation for $\int_0^6 f(x)dx$, found with 3 subintervals of equal length is 52. What is the value of k?

$\frac{1}{2}(2)(4+k) + \frac{1}{2}(2)(k+8) + \frac{1}{2}(2)(8+12)$

$4 + k + k + 8 + 20 = 52$

$2k + 32 = 52$

$2k = 20$

$\boxed{k = 10}$

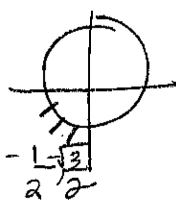
An object moves along the x-axis with initial position x(0) = 2. The velocity of the object is v(t) = sin(πt/3). What is the acceleration of the object at t = 4?

$v(t) = \sin(\pi t/3) = \sin\left(\frac{\pi}{3}t\right)$

$a(t) = v'(t) = \cos\left(\frac{\pi}{3}t\right)\left(\frac{\pi}{3}\right)$

$a(4) = \cos\left(\frac{\pi}{3}(4)\right)\left(\frac{\pi}{3}\right)$

$-\frac{1}{2}\left(\frac{\pi}{3}\right) = \boxed{-\frac{\pi}{6}}$




$$\int_1^4 \frac{u-2}{\sqrt{u}}\,du$$

$u^{1/2}$

$$\int_1^4 u^{1/2} - 2u^{-1/2}\,du$$

$$\frac{u^{3/2}}{\frac{3}{2}} - \frac{2u^{1/2}}{1/2}\Big]_1^4$$

$$\frac{2}{3}u^{3/2} - 4u^{1/2}\Big]_1^4$$

$$\left[\frac{2}{3}(4)^{3/2} - 4(4)^{1/2}\right] - \left[\frac{2}{3}(1)^{3/2} - 4(1)^{1/2}\right]$$

$$\frac{16}{3} - 8 - \frac{2}{3} + 4 = \boxed{\frac{2}{3}}$$

Approximate $\int_0^{\pi} (2x\sin x)\ dx$ using four subintervals of equal length and a Right Hand Riemman sum. (Calculator)

$2\left(\frac{3\pi}{4}\right)\left(\frac{\sqrt{2}}{2}\right)$

$2\pi \sin\pi$

$2\frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right)$

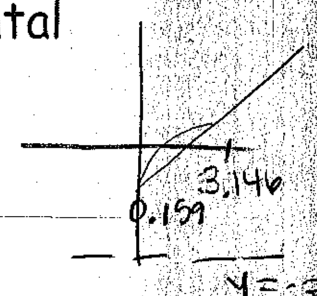
$2\frac{\pi}{2}(1)$

| 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|
| 0 | $\frac{\sqrt{2}\pi}{4}$ | $\pi$ | $\frac{3\pi\sqrt{2}}{4}$ | 0 |

$$\frac{\pi}{4}\left(\frac{\sqrt{2}\pi}{4}\right) + \left(\frac{\pi}{4}\right)(\pi) + \left(\frac{\pi}{4}\right)\left(\frac{3\pi\sqrt{2}}{4}\right) + \left(\frac{\pi}{4}\right)(0)$$

$$0.872 + 2.467 + 2.617 + 0 = \boxed{5.956}$$

Let R be the region bounded by y = lnx and y = x-2. Find the volume of the solid generated when R is rotated about the horizontal line y = -3.
(Calculator)




$$V = \pi\int_{0.159}^{3.146} [\ln x + 3]^2 - [x - 2 + 3]^2\,dx$$

$\boxed{34.199}$