AP Calculus AB
Final Exam Review

Group Challenge (Units 3 & 4)

Find the limit:

$$\lim_{h \to 0} \frac{\cos 4(x+h) - \cos 4x}{h}$$

$f(x) = \cos 4x$

$f'(x) = -\sin(4x)(4)$

$= \boxed{-4\sin(4x)}$

Find the derivative of

$H(x) = e^{x\ln x}$ ↘ Product Rule

$H'(x) = e^{x\ln x}\left[(1)(\ln x) + (x)\left(\frac{1}{x}\right)\right]$

$= \boxed{e^{x\ln x}\left[\ln x + 1\right]}$

Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

inv.

$y = (2x+1)^3$

$x = (2y+1)^3$

$\sqrt[3]{x} = 2y+1$

$\sqrt[3]{x} - 1 = 2y$

$\frac{\sqrt[3]{x}-1}{2} = g(x)$

$g(x) = \frac{1}{2}x^{1/3} - \frac{1}{2}$

$g'(x) = \frac{1}{6}x^{-2/3}$

$g'(1) = \frac{1}{6}x^{-2/3}$

$\frac{1}{6}(1)^{-2/3} = \boxed{\frac{1}{6}}$

Find the derivative of $\cos^2(4x^2 + 7x)$

$[\cos(4x^2+7x)]^2$

$2[\cos(4x^2+7x)]' [\sin(4x^2+7x)] \cdot (8x+7)$

$\boxed{-(16x+14)\cos(4x^2+7x)\sin(4x^2+7x)}$

Consider the closed curve in the xy-plane given by $x^2 + 2x + y^4 + 4y = 5$. Write an equation for the line tangent to the curve at the point (-2, 1).

$2x + 2 + 4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(4y^3+4) = -2x + -2$

$\frac{dy}{dx} = \frac{-2x-2}{(4y^3+4)}$

$\text{slope} = \frac{-2(-2)-2}{4(1)^3+4} = \frac{2}{8} = \frac{1}{4}$

$\boxed{y-1 = \frac{1}{4}(x+2)}$

Where is the point of inflection of g(x) if $g(x) = xe^{2x}$?

$g'(x) = (1)(e^{2x}) + (x)(e^{2x})(2)$

$e^{2x} + 2xe^{2x}$

$g''(x) = 2e^{2x} + (2)(e^{2x}) + (2x)(2e^{2x})$

$4e^{2x} + 4xe^{2x}$

$4e^{2x}(x+1) = 0 \quad x=-1$

$\boxed{\text{P.O.I } x=-1}$

Find the derivative of $(x^2 + 2)(x^2 - 3x)$ Prod. Rule

$(2x)(x^2-3x) + (x^2+2)(2x-3)$

$2x^3 - 6x^2 + 2x^3 - 3x^2 + 4x - 6$

$\boxed{4x^3 - 9x^2 + 4x - 6}$

$V = \frac{1}{3}\pi r^2 h$

A conical tank with the vertex down is 10 feet across the top and 12 ft. deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.

$\frac{r}{h} = \frac{5}{12}$

$12r = 5h$

$r = \frac{5h}{12}$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{5h}{12}\right)^2 (h) = \frac{1}{3}\pi\left(\frac{25}{144}\right)h^3$

$= \frac{25}{432}\pi h^3$

$\frac{dV}{dt} = \frac{75}{432}\pi h^2 \frac{dh}{dt}$

$10 = \frac{75}{432}\pi (8)^2 \frac{dh}{dt}$

$dh/dt = \boxed{\frac{9}{10\pi} \text{ ft/min}}$

$\boxed{y = \frac{1}{4}(x-1)}$

Find the equation of the tangent line to the graph of $y = \frac{\ln x}{4x}$ when $x = 1$

P.O.T →

$y = \frac{\ln 1}{4(1)} = \frac{0}{4} = 0$

$(1,0)$

S.O.T. $f'(x) = \frac{(4x)(\frac{1}{x}) - (\ln x)(4)}{(4x)^2}$

$f'(1) = \frac{4 - (\ln 1)(4)}{(4(1))^2} = \frac{4}{16} = 1/4$

Use the 2nd Derivative Test to find the relative maximum of $g(x) = 3x - x^3 + 5$.

$g'(x) = 3 - 3x^2 = 0$

$-3(x^2 - 1) = 0$

$x = 1 \quad x = -1$

$\boxed{x = 1 \text{ rel. max}}$

$(1,0)$

$g''(x) = -6x$

$g''(-1) = -6(-1) = + \quad$ conc ↑, rel min

$g''(1) = -6(1) = - \quad$ conc ↓, rel. max

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−2	1	−1	2	4
−1	3	−2	1	1
0	−1	2	−2	−3

If $J(x) = \frac{3x + \cos x}{f(x)}$

what is the value of $J'(0)$?

$J'(x) = \frac{f(x)[3 - \sin x] - [3x + \cos x] f'(x)}{[f(x)]^2}$

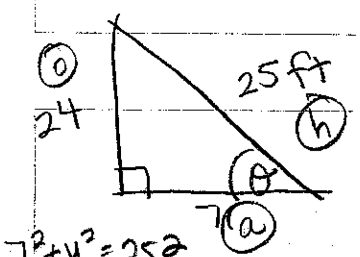
$J'(0) = \frac{f(0)[3 - \sin 0] - [3(0) + \cos 0] f'(0)}{[f(0)]^2}$

$= \frac{(-1)(3-0) - (0+1)(2)}{(-1)^2}$

$= \frac{-3-2}{1}$

$= \boxed{-5}$

A ladder is 25 feet long and is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. Find the rate at which the angle between the ladder and the ground is changing when the base of the ladder is 7 feet from the wall.



$7^2 + y^2 = 25^2$

$\cos\theta = \frac{x}{25}$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$

$-\left(\frac{24}{25}\right)\frac{d\theta}{dt} = \left(\frac{1}{25}\right)(2)$

$\boxed{-1/12 \text{ rad/sec}}$

Find the derivative of $F(x) = x^5e^{3x}$

$F'(x) = (5x^4)(e^{3x}) + (x^5)(e^{3x})(3)$

$= 5x^4e^{3x} + 3x^5e^{3x}$

$= \boxed{x^4e^{3x}(5+3x)}$

Find $\frac{d^2y}{dx^2}$ given the curve $y^2 = x^2 + 2x$.

$2y\frac{dy}{dx} = 2x + 2$

$\frac{dy}{dx} = \frac{2x+2}{2y} = \frac{x+1}{y}$

$\frac{d^2y}{dx^2} = \frac{(y)(1) - (x+1)\left(\frac{dy}{dx}\right)}{(y)^2} = \frac{(y) - (x+1)\left(\frac{x+1}{y}\right)}{y^2}$

$= \boxed{\frac{y^2 - (x+1)^2}{y^3}}$

Find the slope of the normal line to the graph of $f(x) = \sin^2 x$ when $x = (3\pi)/4$.

$-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$

$f(x) = (\sin x)^2$

$f'(x) = 2(\sin x)^1(\cos x)$

$f'\left(\frac{3\pi}{4}\right) = 2\left(\sin\frac{3\pi}{4}\right)\left(\cos\frac{3\pi}{4}\right)$

$= \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{4}}{2} = -\frac{2}{2} = -1$

$\boxed{\text{normal slope} = 1}$

If f(x) = csc3x, then what is f'(x)?

$f'(x) = -\csc(3x)\cot(3x)(3)$

$\boxed{-3\csc(3x)\cot(3x)}$

Find g'(x) if $g(x) = \sqrt{2x+5}$

$g(x) = (2x+5)^{1/2}$

$g'(x) = \cancel{\tfrac{1}{2}}(2x+5)^{-1/2}(\cancel{2})$

$= \boxed{\dfrac{1}{\sqrt{2x+5}}}$

$\boxed{y + \frac{5}{2} = \frac{-1}{4}(x+1)}$

Find the equation of the line tangent to the graph of $g(x) = \dfrac{2x^2 - 3x}{3x + 1}$ when x = -1

$(-1, -5/2)$

P.O.T → $g(-1) = \dfrac{2(-1)^2 - 3(-1)}{3(-1)+1} = \dfrac{5}{-2}$

S.O.T → $g'(x) = \dfrac{(3x+1)(4x-3) - (2x^2-3x)(3)}{(3x+1)^2}$

$g'(-1) = \dfrac{(-2)(-7) - (5)(3)}{4} = -\frac{1}{4}$

Find the slope of the normal line drawn to the graph of g(x) = xsinx when x = π

$(-1, 0)$

S.O.T → $g'(x) = (1)(\sin x) + (x)(-\cos x)$

$g'(\pi) = \sin\pi + -\pi(\cos\pi)$

$= 0 + -\pi(-1)$

$= \pi$

S.O.N $= \boxed{\frac{1}{\pi}}$

On what interval is g(x) concave up if $g'(x) = \sqrt{8x - x^2}$

$g'(x) = (8x - x^2)^{1/2}$

$g''(x) = \frac{1}{2}(8x - x^2)^{-1/2}(8 - 2x)$ $f''(x)$

$g''(x) = \dfrac{4 - x}{\sqrt{8x - x^2}}$

$4 - x = 0$
$x = 4$

$\sqrt{8x - x^2} = 0$
$8x - x^2 = 0$
$-x(x - 8) = 0$
$x = 0 \quad x = 8$

0 4 8
undef + pos − neg undef

$\boxed{\text{conc.} \uparrow (0,4)}$

x	f(x)	f'(x)	g(x)	g'(x)
−2	1	−1	2	4
−1	3	−2	1	1
0	−1	2	−2	−3

If J(x) = g(x)sinx, what is the value of J'(0)?

(0,0)

$J'(x) = g'(x)(\sin x) + (g(x)(\cos x)$

$J'(0) = g'(0)\sin 0 + g(0)\cos 0$

$= (-3)(0) + (-2)(1) = \boxed{-2}$

Find dy/dx of $2x^3 - y^2 = 3y$

$6x^2 - 2y\frac{dy}{dx} = 3\frac{dy}{dx}$

$\frac{dy}{dx}(-2y - 3) = -6x^2$

$\frac{dy}{dx} = \boxed{\dfrac{-6x^2}{-2y-3} \text{ or } \dfrac{6x^2}{2y+3}}$

For what values of a and b will the function below be differentiable at x = 1?

$$f(x) = \begin{cases} 3ax^2 + 2bx + 1, & x \le 1 \\ ax^4 - 4bx^2 - 3x, & x > 1 \end{cases}$$

$f'(x) = \begin{cases} 6ax + 2b, & x \le 1 \\ 4ax^3 - 8bx - 3, & x > 1 \end{cases}$

$6a(1) + 2b = 4a(1)^3 - 8b(1) - 3$

$6a + 2b = 4a - 8b - 3$

$2a + 10b = -3$

$-2a - 6b = +4$

$4b = 1 \quad \boxed{b = 1/4}$

$3a(1)^2 + 2b(1) + 1 = a(1)^4 - 4b(1)^2 - 3(1)$

$3a + 2b + 1 = a - 4b - 3$

$2a + 6b = -4$

$\boxed{a = -11/4}$