(no calc.) AP Calculus AB - Scav. Hunt (Units 1 & 3)

J.

$$\lim_{x \to 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x}$$

$$\frac{(2)\,1}{(2)\,x+2} + \frac{-1\,(x+2)}{2\,(x+2)} = \frac{\cancel{2}-x-\cancel{2}}{2(x+2)} = \frac{-x}{2(x+2)}$$

$$\frac{-\cancel{x}}{2(x+2)} \cdot \frac{1}{\cancel{x}}$$

$$\lim_{x \to 0} \frac{-1}{2(x+2)} = \frac{-1}{2(0+2)} = \boxed{\frac{-1}{4}}$$ ✓

When is $f(x) = 2x^3 + 3x^2 - 12x$ increasing?

A.

$$f'(x) = 6x^2 + 6x - 12$$
$$= 6(x^2 + x - 2)$$
$$(x+2)(x-1)$$

$x = -2 \quad x = 1$

-3: $(-)(-)$ pos; -2; 0: $(+)(-)$ neg; $+1$; (2): $(+)(+)$ pos

$\boxed{(-\infty, -2) \cup (1, \infty)}$ ✓

Q.

$$\lim_{x \to 3^-} \frac{x+3}{x-3} \qquad \frac{2.9+3}{2.9-3} = \frac{\oplus}{\ominus}$$

Plug in 2.9

$\boxed{-\infty}$ ✓

I.

$a = x \quad b = 2$

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}}$$

$$\lim_{x \to 2} (x^2 + 4x + 4) =$$

$$(2)^2 + 2(2) + 4 =$$
$$4 + 4 + 4 = \boxed{12}$$ ✓

H.

If $f(x) = 2x^2 + 1$, then find

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x^2}$$

$f(0) = 2(0)^2 + 1$

$$\lim_{x \to 0} \frac{2x^2 + \cancel{1} - \cancel{1}}{(x)^2}$$

$$\lim_{x \to 0} \frac{2\cancel{x^2}}{\cancel{x^2}} = \lim_{x \to 0} 2 = \boxed{2}$$ ✓

K.

Determine the values of x at which the function $f(x) = x^4 - 8x^2 + 2$ has a horizontal tangent.

$f'(x) = 4x^3 - 16x$

$4x(x^2 - 4) = 0$

$x = 0 \quad x^2 = 4$

$x = 2 \quad x = -2$

$\boxed{-2, 0, 2,}$ ✓

P.

$$\lim_{x \to -\infty} \frac{5x + 1}{\sqrt{x^2} - x}$$

$\sqrt{x^2} = -x$

$$\lim_{x \to -\infty} \frac{\frac{5x}{-x} + \frac{1}{-x}}{\sqrt{\frac{x^2}{x^2} - \frac{x}{x^2}}}$$

$$\lim_{x \to -\infty} \frac{-5 - \frac{1}{x}}{\sqrt{1 - \frac{1}{x}}} = \frac{-5 - 0}{\sqrt{1 - 0}} = \frac{-5}{\sqrt{1}} = \boxed{-5}$$ ✓

B.

$$\lim_{x \to 0} \frac{3 - 3\cos x}{x}$$

$$\lim_{x \to 0} \frac{3(1 - \cos x)}{x}$$

$= 3(0) = \boxed{0}$ ✓

G.

$$\lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x - 1}$$

$$\frac{\sqrt{x+3}-2}{x-1} \cdot \frac{(\sqrt{x+3}+2)}{\sqrt{x+3}+2} = \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$$

$$= \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x+3}+2)}$$

$$\lim_{x \to 1} \frac{1}{(\sqrt{x+3}+2)} = \frac{1}{\sqrt{1+3}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}} \checkmark$$

R. Find the value of a that makes the function continuous everywhere.

$$f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$$

$$4-(-1)^2 = a(-1)^2 - 1$$

$$4 - 1 = a - 1$$

$$a = \boxed{4} \checkmark$$

C. Find the relative maximum of $f(x) = 2x^3 + 3x^2 - 12x$.

$(+) \to (-)$

$$f'(x) = 6x^2 + 6x - 12$$

$$= 6(x^2 + x - 2)$$

$$= 6(x+2)(x-1)$$

$x = -2 \quad x = 1$

$-3 \quad -2 \quad 0 \quad 1 \quad 2$

$(-)(-)$ pos $\quad (+)(-)$ neg $\quad (+)(+)$ pos

$\boxed{x = -2}$

L. Find the equation of the normal line drawn to the graph of $f(x) = x^4 - 2x$ at $x = 0$.

point $(0, 0)$

$(0)^4 - 2(0)$

$$f'(x) = 4x^3 - 2$$

$$f'(0) = 4(0)^3 - 2$$

Slope of tangent $= -2$

normal slope $= 1/2$

$$y - 0 = \frac{1}{2}(x - 0) \qquad \boxed{y = \frac{1}{2}x}$$

M.

$$\lim_{x \to 0} \frac{6x + \sin x}{x}$$

$$\lim_{x\to 0} \frac{6\cancel{x}}{\cancel{x}} + \lim_{x \to 0} \frac{\sin x}{x}$$

$$\lim_{x\to 0} 6 + \lim_{x\to 0} \frac{\sin x}{x}$$

$$6 + 1 = \boxed{7}$$

S.

$$\lim_{x \to 3} f(x) \text{ when}$$

$$f(x) = \begin{cases} 2x^2 - 3x, & x < 3 \\ 8 - \cos(\pi x/3), & x > 3 \end{cases}$$

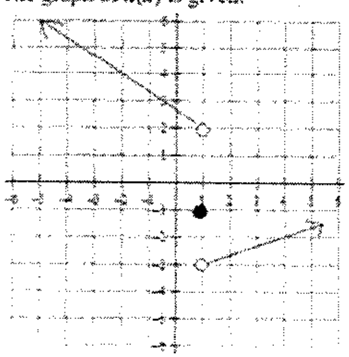
$$\lim_{x \to 3^-} f(x) = 2(3)^2 - 3(3) = 9$$

$$\lim_{x\to 3^+} f(x) = 8 - \cos\left(\frac{3\pi}{3}\right) = 9 \qquad \boxed{9}$$

$$8 - (-1)$$

F.

The graph of $h(x)$ is given.



$$\lim_{x \to 1^+} h(x) = \boxed{-3}$$

D.

$$\lim_{x \to 2} \frac{\sqrt{x+2} - 2}{2x^2 - 8}$$

$$\left(\frac{\sqrt{x+2} - 2}{2x^2-8}\right)\left(\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}\right) = \frac{x+2-4}{(2x^2-8)(\sqrt{x+2}+2)}$$

$$= \frac{(x-2)}{2(x^2-4)(\sqrt{x+2}+2)}$$

$$= \frac{\cancel{(x-2)}}{2(x+2)\cancel{(x-2)}(\sqrt{x+2}+2)}$$

$$\lim_{x\to 2} \frac{1}{2(x+2)(\sqrt{x+2}+2)}$$

$$= \frac{1}{2(2+2)(\sqrt{2+2}+2)}$$

$$= \frac{1}{2(4)(4)} = \boxed{\frac{1}{32}}$$

D.

When is $f(x) = x^3 - 6x^2 + 15$ decreasing?

$f'(x) = 3x^2 - 12x$

$3x(x-4) = 0$

$x = 0 \quad x = 4$

-1 | 2 | 5

$(-)(-)$ pos | 0 | $(+)(-)$ neg | 4 | $(+)(+)$ pos

$\boxed{(0,4)}$

E.

$$\lim_{x \to 0} \frac{\cos 4x \tan 4x}{6x}$$

$\cancel{\cos 4x}\left(\frac{\sin 4x}{\cancel{\cos 4x}}\right)$

$$\lim_{x \to 0} \frac{\sin 4x}{6x} = \frac{4}{6} \lim_{x \to 0} \frac{\sin 4x}{4x}$$

$$= \frac{4}{6}(1) = \boxed{\frac{2}{3}}$$

N.

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

means find deriv of $\sqrt{x}$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$= \boxed{\frac{1}{2\sqrt{x}}}$

T.

Find the equation of the line tangent to the graph of $f(x) = x^4 + 3x$ at $x = -1$.

$f'(x) = 4x^3 + 3$

point $(-1, -2)$

$(-1)^4 + 3(-1)$

$f'(-1) = 4(-1)^3 + 3$

$= -1 \leftarrow$ slope of tangent

$\boxed{y + 2 = -(x+1)}$