AP Calculus AB
Unit 3 - REVIEW

Name: Answer Key*

NO CALCULATOR PERMITTED	
1. B	If $y = x\sin x$, then $\frac{dy}{dx} =$ Product Rule $(1)(\sin x) + (x)(\cos x$ $\sin x + x\cos x$ (A) $\sin x + \cos x$ (B) $\sin x + x\cos x$ (C) $\sin x - x\cos x$ (D) $x(\sin x + \cos x)$ (E) $x(\sin x - \cos x)$
2. E	If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$ $f'(x) = 7 + \frac{1}{x}$ $f'(1) = 7 + \frac{1}{1} = 8$ (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
3. E	If $y = (x^3 - \cos x)^5$, then $y' =$ Chain Rule $5(x^3 - \cos x)^4 (3x^2 + \sin x)$ (A) $5(x^3 - \cos x)^4$ (B) $5(3x^2 + \sin x)^4$ (C) $5(3x^2 + \sin x)$ (D) $5(3x^2 + \sin x)^4 \cdot (6x + \cos x)$ (E) $5(x^3 - \cos x)^4 \cdot (3x^2 + \sin x)$
4. A	If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is (A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$ $f(g(x)) = \sqrt{(3x-2)^2 - 4} = \sqrt{9x^2 - 12x + 4 - 4} = \sqrt{9x^2 - 12x} = (9x^2 - 12x)^{1/2}$

$[f(g(x))]' = \frac{1}{2}(9x^2 - 12x)^{-1/2}(18x - 12)$

$= \frac{9x - 6}{\sqrt{9x^2 - 12x}} = \frac{9(3) - 6}{\sqrt{9(3)^2 - 12(3)}} = \frac{21}{\sqrt{45}} = \frac{21}{3\sqrt{5}} = \frac{7}{\sqrt{5}}$

5. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

C

(A) (0,0) only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) (0,0) and (−4,2)

(D) (0,0) and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

$x + 2 \quad -x$

$f'(x) = \frac{(x+2)(1) - (x)(1)}{(x+2)^2} = \frac{2}{(x+2)^2}$

$\frac{2}{(x+2)^2} = \frac{1}{2}$

$\pm\sqrt{4} = \sqrt{(x+2)^2}$

$\pm 2 = x + 2$

$-2 \pm 2 = x$

$x = -4, \quad x = 0$

$\left(-4, \frac{-4}{-4+2}\right) \quad \left(0, \frac{0}{0+2}\right)$

$(-4, -2) \quad (0, 0)$

6. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

D

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

$y = (2x+1)^3$

$x = (2y+1)^3$

$\sqrt[3]{x} = 2y + 1$

$g(x) = \frac{\sqrt[3]{x} - 1}{2}$

$g(x) = \frac{1}{2}x^{1/3} - \frac{1}{2}$

$g'(x) = \frac{1}{6}x^{-2/3} = \frac{1}{6\sqrt[3]{x^2}}$

$g'(1) = \frac{1}{6\sqrt[3]{1^2}} = \frac{1}{6}$

7. The $\lim_{h \to 0} \frac{\ln[\sin(x+h)] - \ln(\sin x)}{h}$ is...

D

$f(x) = \ln(\sin x)$

$f'(x) = \frac{\cos x}{\sin x} = \cot x$

A. $\sin x$ B. x C. $\frac{1}{x}$ D. $\cot x$

8. The $\lim_{x \to \frac{\pi}{2}} \frac{\sin(x) - \sin\left(\frac{\pi}{2}\right)}{x - \frac{\pi}{2}}$ has a value of ...

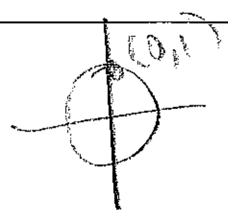
$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ Derivative at $x = a$

A

A. 0 B. 1 C. $\frac{\sqrt{2}}{2}$ D. −1

$f(x) = \sin x \quad a = \pi/2$

$f'(x) = \cos x$



$f'(\pi/2) = \cos(\pi/2) = 0$

↙ Derivative

9. The equation of the normal line to the graph of $y = e^{2x}$ when $\frac{dy}{dx} = 2$ is… point on normal →

A

A. $y = -\frac{1}{2}x + 1$

B. $y = 2\left(x - \frac{\ln 2}{2}\right) + 2$

C. $y = 2x + 1$

D. $y = -\frac{1}{2}\left(x - \frac{\ln 2}{2}\right) + 2$

$\frac{dy}{dx} = e^{2x}(2)$

$2e^{2x} = 2$

$e^{2x} = 1$

$\cancel{\ln e^{2x}} = \ln 1$

$2x = 0$

$x = 0$

$y = e^{2(0)}$

$y = 1 \quad (0,1)$

S.O.N $= -1/2$

$\boxed{y - 1 = -\frac{1}{2}x}$

$y = -1/2x + 1$

10. If $f(x) = 5\cos^2(\pi - x)$, then $f'\left(\frac{\pi}{2}\right)$ is …

A

A. 0

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. $-\frac{5}{6}$

$f(x) = 5[\cos(\pi - x)]^2$

$f'(x) = 10[\cos(\pi - x)]^1 \left(-\sin(\pi - x)(-1)\right)$

$-10[\cos(\pi - x)][\sin(\pi - x)]$

$f'\left(\frac{\pi}{2}\right) = -10\left[\cos\left(\pi - \frac{\pi}{2}\right)\right]\left[\sin(\pi - \pi/2)\right]$

$-10\left[\cos\frac{\pi}{2}\right][\sin \pi/2] = -10(0)(1) = 0$

11. For what value(s) of k does the graph of $g(x) = ke^{2x} + 3x$ have a normal line whose slope is $-\frac{1}{5}$ when $x = 1$?

B

slope of tangent = 5

$g'(x) = ke^{2x}(2) + 3$

$g'(1) = ke^{2(1)}(2) + 3$

A. e B. $\frac{1}{e^2}$ C. $-\frac{8}{5e^2}$ D. $\frac{2}{e^2}$

$2ke^2 + 3 = 5$

$2ke^2 = 2$

$ke^2 = 1 \quad k = \frac{1}{e^2}$

12. If $f'(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

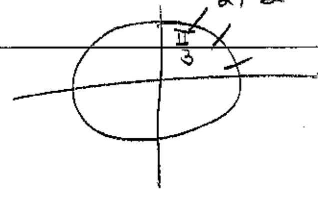
$f'\left(\frac{\pi}{6}\right) = \tan\left(2\left(\frac{\pi}{6}\right)\right) = \tan\left(\frac{\pi}{3}\right)$

C

A. $2\sqrt{3}$ B. 4 C. $\sqrt{3}$ D. 8

$\frac{1}{2}, \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$



f(x) = **CALCULATOR PERMITTED**

13. The graph of ~~y~~ f $= e^{\tan x} - 2$ crosses the x-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747

D

$e^{\tan x} - 2 = 0$ $\quad e^{\tan x}(\sec^2 x)$

$x = .6061119$

(Find x-int.) $\quad f'(.6061119) = e^{\tan(.6061119)} \sec^2(.6061119) = 2.961$

14. Given that $f(x) = x^2 e^x$, what is an approximate value of $f(1.1)$ if you use the equation of the tangent line to the graph of f at $x = 1$?

$f(1) = (1)^2 e^1 = (1, e)$ ← point

$f'(x) = (2x)(e^x) + (x^2)(e^x)(1)$

$f'(1) = 2(1)e^1 + (1)^2 e^1(1)$

$= 2e + e = 3e$ ← slope of tangent

A. 3.534

B. 3.635

C. 7.055

D. 8.155

A

$y - e = 3e(x-1)$

$y - e = 3e(1.1-1) \rightarrow y = 3.534$

15. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangents?

C

A. −0.701 B. −0.567 C. −0.391 D. −0.302

$f'(x) = 3e^{2x}(2) = 6e^{2x}$ $\quad g'(x) = 18x^2$

$\underbrace{6e^{2x}}_{y_1} = \underbrace{18x^2}_{y_2}$ $\quad x = -.391$

Find where they intersect.

16. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$? ← slope of tangent

D

~~A.~~ $y = 8x - 5$ B. $y = x + 7$ C. $y = x + 0.763$ D. $y = x - 0.122$

$f'(x) = 4x^3 + 4x$

$\underbrace{4x^3 + 4x}_{y_1} = \underbrace{1}_{y_2}$

$f(0.237) = (.237)^4 + 2(.237)^2$

$= 0.115$

$(.237, 0.115)$ point

$x = 0.237$ $\quad y - .115 = 1(x - .237)$

$y = x - .122$

FREE RESPONSE #1 (calc. Inactive)

Consider the piece-wise defined function below to answer the questions that follow.

$$f(x)=\begin{cases} ax^2+bx+2, & x\le 2 \\ ax+b, & x>2 \end{cases}$$

a. If $a = -3$ and $b = 4$, will $f(x)$ be continuous at $x = 2$? Justify your answer.

$f(x)=\begin{cases} -3x^2+4x+2, & x\le 2 \\ -3x+4, & x>2 \end{cases}$

① $f(2) = -3(2)^2+4(2)+2 = -2$
$f(2)$ is defined ✓

② $\lim\limits_{x\to 2^-} f(x) = -3(2)^2+4(2)+2 = -2$
$\lim\limits_{x\to 2^+} f(x) = -3(2)+4 = -2$
So, $\lim\limits_{x\to 2} f(x)$ exists. ✓

③ $f(2) = \lim\limits_{x\to 2} f(x) = -2$ ✓

$\therefore$ $f(x)$ is continuous at $x=2$.

b. If $a = -3$ and $b = 4$, will $f(x)$ be differentiable at $x = 2$? Justify your answer.

① $f(x)$ is continuous at $x=2$ (see part a.)

② $f'(x)=\begin{cases} -6x+4, & x\le 2 \\ -3, & x>2 \end{cases}$

$\lim\limits_{x\to 2^-} f'(x) = -6(2)+4 = -8$
$\lim\limits_{x\to 2^+} f'(x) = -3$
$\lim\limits_{x\to 2} f'(x)$ D.N.E

$\therefore$ $f(x)$ is not differentiable at $x=2$.

c. For what value(s) of a and b will $f(x)$ be both continuous and differentiable at $x = 2$? Show your work.

For $f(x)$ to be continuous,

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$

$a(2)^2+b(2)+2 = a(2)+b$

$4a+2b+2 = 2a+b$

$2a+b=-2$

$2a+-3a=-2$

$-a=-2$

$\boxed{a=2}$

For $f(x)$ to be diff,

$f'(x)=\begin{cases} 2ax+b, & x\le 2 \\ a, & x>2 \end{cases}$

$\lim\limits_{x\to 2^-} f'(x) = \lim\limits_{x\to 2^+} f'(x)$

$2a(2)+b = a$

$4a+b=a$

$b=-3a$

$b=-3(2)$

$\boxed{b=-6}$

FREE RESPONSE #2 (calc. Active)

A rodeo performer spins a lasso in a circle perpendicular to the ground. The height from the ground of the knot, measured in units of feet, in the lasso is modeled by the function

$$H(t) = -3\cos\left(\frac{5\pi}{3}t\right) + 5,$$

where t is the time measured in seconds after the lasso begins to spin.

a. Find the value of $H(0.75)$. Using correct units, explain what this value represents in the context of this problem.

$H(0.75) = -3\cos\left(\frac{5\pi}{3}(.75)\right) + 5 = \boxed{7.121}$

0.75 seconds after the lasso begins to spin, the height of the knot from the ground is 7.121 feet.

b. Find the value of $H'(0.75)$. Using correct units, explain what this value represents in the context of this problem.

use math 8 * on calc

$H'(0.75) \Rightarrow \frac{d}{dx}\left(-3\cos\left(\frac{5\pi}{3}t\right)+5\right)\Big|_{x=.75} = \boxed{-11.107}$

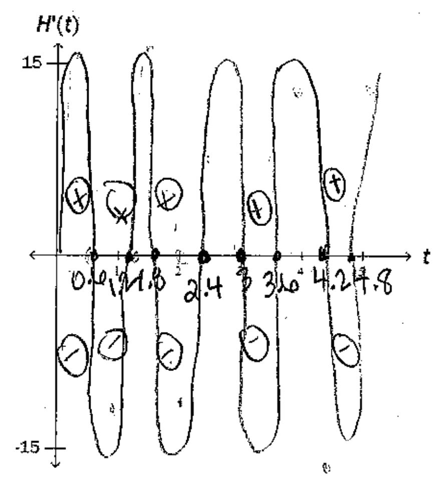
At 0.75 seconds after the lasso begins to spin, the height is decreasing at a rate of 11.107 feet per second.

c. Find $H'(t)$ and sketch its graph on the axes to the right for the interval $0 < t < 5$ seconds.

$H(t) = -3\cos\left(\frac{5\pi}{3}t\right) + 5$

$H'(t) = \cancel{+3}\cdot\cancel{-}\sin\left(\frac{5\pi}{3}t\right)\cdot\frac{5\pi}{\cancel{3}}$

$\boxed{H'(t) = 5\pi\sin\left(\frac{5\pi}{3}t\right)}$



H'(t)

d. During the first five seconds of the performer spinning the lasso, how many times is the lasso at its maximum height? Give a reason for your answer based on the graph of $H'(t)$.

max height → on x-intercepts & changes from + to −

$\boxed{\text{4 Max heights}}$

e. What is the height of the lasso the first time it is at its minimum height on the interval $0 < t < 5$ seconds? Justify your answer and show your work.

min height → on x-intercepts & changes from − to +

$\boxed{X=1.2}$