* Calc. Permitted

AP Calculus AB
Unit 3 – Day 6 – Assignment

Name: Answer Key*

MULTIPLE CHOICE

1. Which of the following statements can be made about the graph of the function $h(x) = \dfrac{\ln(\cos x)}{\tan x}$ when $x = \frac{\pi}{2}$.

$\tan\frac{\pi}{2} = \text{undef.}$

$$h'(x) = \frac{(\tan x)\left(\frac{-\sin x}{\cos x}\right) - (\ln(\cos x))(\sec^2 x)}{(\tan x)^2}$$

$$\frac{-\tan^2 x - \sec^2 x(\ln(\cos x))}{(\tan x)^2}$$

$$\frac{-(\tan(\frac{\pi}{2}))^2 - (\sec(\frac{\pi}{2}))^2(\ln\cos\frac{\pi}{2})}{(\tan(\frac{\pi}{2}))^2}$$

$\tan(\frac{\pi}{2})$

$\frac{y}{x} = \frac{1}{0}$

A. The graph of $h(x)$ is increasing.
B. The graph of $h(x)$ is decreasing.
C. No conclusion can be made about the graph of $h(x)$.
D. The graph of $h(x)$ has a horizontal tangent.

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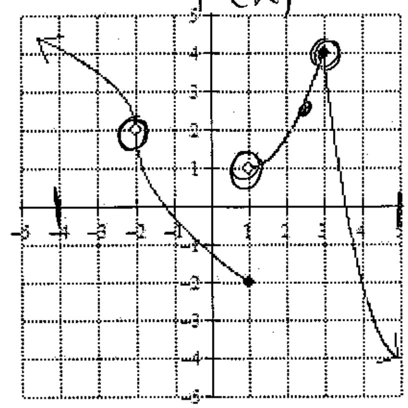
2. Consider the graph of $f(x)$ to the right to determine which of the following statements is/are true.

~~I.~~ $f'(x) = 0$ when $x = 3$. $f'(x) = \text{undef}$ b/c cusp!

II. $f'(2.5) > 0$. yes, increasing

III. On the interval $(-4, 5)$ there are three values of x at which $f(x)$ is not differentiable.

f(x)



A. II and III only
B. I and II only
C. III only
D. I, II and III

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3. Let $f(7) = 0$, $f'(7) = 14$, $g(7) = 1$ and $g'(7) = \frac{1}{7}$. Find $h'(7)$ if $h(x) = \dfrac{f(x)}{g(x)}$.

A. 98
B. –14
C. –2
D. 14

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$h'(7) = \frac{g(7)f'(7) - f(7)g'(7)}{(g(7))^2}$$

$$= \frac{(1)(14) - (0)(\frac{1}{7})}{(1)^2} = \frac{14-0}{1} = 14$$

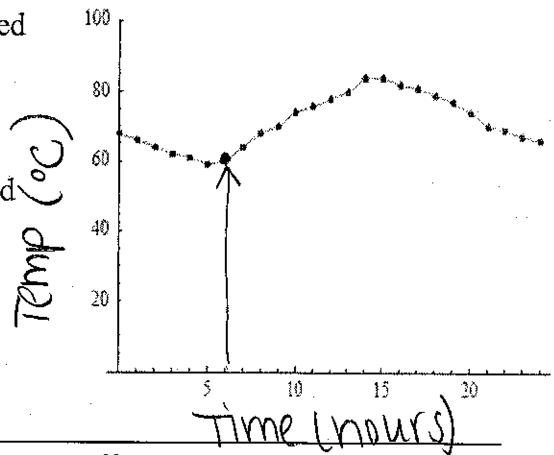
4. The graph to the right shows data of a function, $H(t)$, which shows the relationship between temperature in °C (y-axis) and the time in hours (x-axis). What does the value of $H'(6)$ represent?

~~A.~~ $H'(6)$ represents the temperature after 6 hours measured in °C.

B. $H'(6)$ represents the rate at which the temperature is changing after 6 hours measured in °C.

~~C.~~ $H'(6)$ represents the temperature after 6 hours measured in °C per hour

D. $H'(6)$ represents the rate at which the temperature is changing after 6 hours measured in °C per hour.



5. The graph of $h(x) = 2\sqrt{x+4}+2$ is pictured below.

What is the value of $[h^{-1}(6)]'$?

A. $\frac{1}{4}$

B. 2

C. 4

D. $\frac{1}{2}$

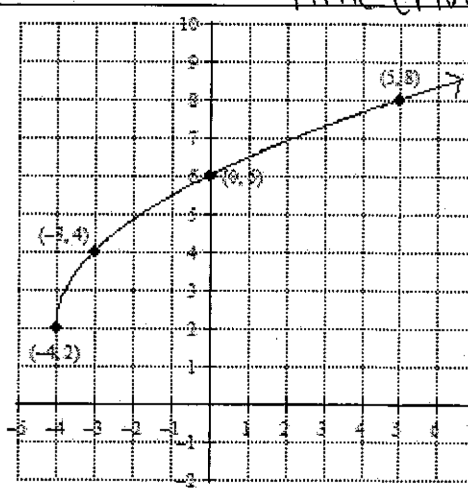
$$[h^{-1}(6)]' = \frac{1}{h'(h^{-1}(6))} = \frac{1}{h'(0)} = \frac{1}{\frac{1}{2}} = 2$$

$h(x) = 2(x+4)^{1/2}+2$

$h'(x) = (x+4)^{-1/2}(1)$

$\frac{1}{\sqrt{x+4}}$

$h'(0) = \frac{1}{\sqrt{0+4}}$



6. Find y' if $y = x^2e^x$.

A. $2xe^x$

B. $x(x+2e^x)$

C. $xe^x(x+2)$

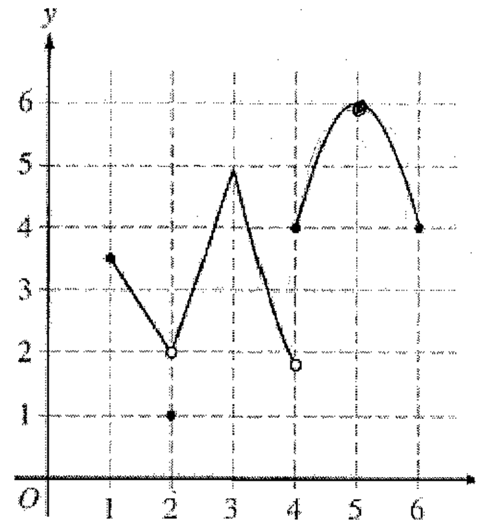
D. $2x+e^x$

$(2x)(e^x) + (x^2)(e^x)(1)$

$2xe^x + x^2e^x$

$xe^x(2+x)$

7. The function f is pictured to the right. At which of the following values of x is f defined and continuous but not differentiable.



Graph of f

~~I.~~ $x = 2$ not cont

II. $x = 3$ cont w| cusp

III. $x = 5$ cont

A. II only
B. I only
C. II and III only
D. I and II only

FREE RESPONSE

The table below shows values of differentiable functions, $f(x)$ and $g(x)$, and their derivatives at selected values of x. Use the table of values below to answer each of the questions below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	−1	2	5
1	3	2	3	−3
2	5	3	1	−2
3	10	4	0	−1

a. Approximate the value of $f'(1.5)$? Explain why your answer is a good approximation of $f'(1.5)$.

(1, 3) (2, 5)

$$f'(1.5) \approx \frac{5-3}{2-1} \approx \frac{2}{1} \approx \boxed{2}$$

f'(1.5) is a good approximation because we used two points close to ~~x~~=1.5 on the secant line. Since the slope of the secant line should be closely parallel, then this approx gives us close to the slope of the tangent line.

b. If $B(x) = \sqrt{g(x)}$, what is the equation of the tangent line drawn to $B(x)$ when $x = 1$?

$B(x) = [g(x)]^{1/2}$

$B'(x) = \frac{1}{2}[g(x)]^{-1/2}[g'(x)]$

$= \frac{1}{2}[g(1)]^{-1/2}[g'(1)]$

$= \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)(-3) = \frac{-3}{2\sqrt{3}}$ → slope of tangent

$B(1) = \sqrt{g(1)} = \sqrt{3}$

$(1, \sqrt{3})$

$\boxed{y - \sqrt{3} = \frac{-3}{2\sqrt{3}}(x-1)}$

c. If $A(x) = x^2 \ln(f(x))$, what is the value of $A'(2)$? What does this result say about the behavior of the graph of $A(x)$ when $x = 2$? Give a reason for your answer.

$A'(x) = (2x)(\ln f(x)) + (x^2)\left(\frac{f'(x)}{f(x)}\right)$

$A'(2) = (2(2))(\ln(f(2))) + (2^2)\left(\frac{f'(2)}{f(2)}\right)$

$= (4)(\ln(5)) + 4\left(\frac{3}{5}\right)$

$= 6.44 + \frac{12}{5} = \boxed{8.84}$

Since $A'(2) > 0$, then the graph of $A(x)$ is increasing when $x = 2$.

d. Find the value of $[g^{-1}(3)]'$. Then, find the equation of the line normal to the graph of $g^{-1}(x)$ at $x = 3$.

$[g^{-1}(3)]' = \frac{1}{g'(g^{-1}(3))} = \frac{1}{g'(1)} = \frac{1}{-3}$ slope of tangent

$g^{-1}(3) = 1$ $(3, 1)$ ← point

slope of normal $= 3$

$\boxed{y - 1 = 3(x-3)}$