AP Calculus AB

Unit 3 - Day 4 WARM-UP

Name: Answer Key*

Find the derivative of each function.

1) cot(5x)

$f'(x) = -\csc^2(5x) \cdot (5)$

$= \boxed{-5\csc^2(5x)}$

2) $\tan(3x^2 + x)$

$f'(x) = \sec^2(3x^2+x) \cdot (6x+1)$

$\boxed{(6x+1)(\sec^2(3x^2+x))}$

3) csc(8x + 1)

$f'(x) = -\csc(8x+1)\cot(8x+1)\,(8)$

$\boxed{-8\csc(8x+1)\cot(8x+1)}$

4) $\sec(9x^2)$

$f'(x) = \sec(9x^2)\tan(9x^2) \cdot (18x)$

$\boxed{18x\sec(9x^2)\tan(9x^2)}$

5) $\sin^2(5x + 1)$

$[\sin(5x+1)]^2$

$2[\sin(5x+1)]^1\,(\cos(5x+1)(5)$

$\boxed{10\sin(5x+1)\cos(5x+1)}$

6) $\cos^2(3x^2 + 5x)$

$[\cos(3x^2+5x)]^2$

$2[\cos(3x^2+5x)]^1\quad(-\sin(3x^2+5x)(6x+5)$

$\boxed{-(12x+10)\cos(3x^2+5x)\sin(3x^2+5x)}$

AP Calculus
Unit 3 – Rules of Differentiation

Day 4 Notes: Finding the Derivative of the Natural Exponential & Logarithmic Functions

Differentiation Rule for Natural Exponential Functions

If $f(x) = e^{g(x)}$, then $f'(x) = e^{g(x)} \cdot g'(x)$

Find the derivative of each of the following functions.

$f(x) = e^{\sin x}$ $f'(x) = e^{\sin x}(\cos x)$ $= \boxed{(\cos x)(e^{\sin x})}$	$f(x) = e^{2x+3}$ $f'(x) = e^{2x+3}(2)$ $= \boxed{2e^{2x+3}}$
$f(x) = 3e^{2x}$ $f'(x) = 3e^{2x}(2)$ $= \boxed{6e^{2x}}$	$f(x) = (2x+3)(e^{3x})$ *Product Rule → $f'(x) = (2)(e^{3x}) + (2x+3)(e^{3x})(3)$ $\underline{2e^{3x}} + 6xe^{3x} + \underline{9e^{3x}}$ $11e^{3x} + 6xe^{3x}$ $\boxed{e^{3x}(6x+11)}$
$f(x) = x^2e^{2x}$ *Product rule → $f'(x) = (2x)(e^{2x}) + (x^2)(e^{2x})(2)$ $= 2xe^{2x} + 2x^2e^{2x}$ $\boxed{2xe^{2x}(x+1)}$	$f(x) = \sqrt{e^{2x-6}}$ $f(x) = (e^{2x-6})^{1/2}$ $f(x) = e^{x-3}$ $f'(x) = e^{x-3}(1)$ $= \boxed{e^{x-3}}$

*Quotient Rule →

$f(x)=\frac{e^{5x}}{3x^2}$ $= \frac{(3x^2)(e^{5x})(5)-(e^{5x})(6x)}{(3x^2)^2}$

$= \frac{15x^2e^{5x}-6xe^{5x}}{9x^4} = \frac{\cancel{3x}e^{5x}(5x-2)}{_3\cancel{9}\cancel{x^4}_3} = \boxed{\frac{e^{5x}(5x-2)}{3x^3}}$

Differentiation Rule for Natural Logarithmic Functions

If $f(x)=\ln[g(x)]$, then $f'(x)=\frac{g'(x)}{g(x)}$.

Find the derivative of each of the following functions.

$f(x)=\ln(2x-3)$ $f'(x)=\boxed{\frac{2}{2x-3}}$	$f(x)=\ln(3x^2+2x)$ $f'(x)=\frac{6x+2}{3x^2+2x}=\boxed{\frac{2(3x+1)}{x(3x+2)}}$
$f(x)=\ln(\cos x)$ $f'(x)=\frac{-\sin x}{\cos x}$ $=\boxed{-\tan x}$	$f(x)=\ln\sqrt{2x-4}$ $\ln(2x-4)^{1/2}$ $f'(x)=\frac{\cancel{\tfrac{1}{2}}(2x-4)^{-1/2}\cancel{(2)}}{(2x-4)^{1/2}}$ $=\frac{1}{(2x-4)^{1/2}(2x-4)^{1/2}}$ $=\boxed{\frac{1}{2x-4}}$

MATH | 8: nDeriv(

Finding Values of Derivatives Using the Graphing Calculator

For each of the functions below, find the value of $f'(x)$ at the indicated value of x using the graphing calculator. Then, determine if the function is increasing, decreasing, has a horizontal tangent or has a vertical tangent. Give a reason for your answer.

Function	Value of $f'(a)$	Is $f(x)$ increasing or decreasing, or does $f(x)$ have a horizontal or a vertical tangent?
1. $f(x) = 3e^x \sin x$	$a = -2$ $f'(-2) = -0.538$	Since $f'(-2) < 0$, then f(x) is decreasing at $x = -2$.
2. $f(x) = 3e^x \sin x$	$a = 1$ $f'(1) = 11.268$	Since $f'(1) > 0$, then f(x) is increasing at $x = 1$.
3. $f(x) = \frac{\ln(\cos x)}{x^2}$	$a = \frac{\pi}{3}$ $f'(\frac{\pi}{3}) = -0.372$	Since $f'(\pi/3) < 0$, then f(x) is decreasing at $x = \pi/3$.
4. $f(x) = \frac{\ln(\cos x)}{x^2}$	$a = \pi$ (Error on CALC) $f'(\pi) =$ undefined	Since $f'(\pi)$ is undefined, then f(x) has a vertical tangent at $x = \pi$.
5. $f(x) = e^{\tan(0.34x)}$	$a = 0$ $f'(0) = 0.340$	Since $f'(0) > 0$, then f(x) is increasing at $x = 0$.
6. $f(x) = 5\sin^2(\ln x)$	$a = 1$ $f'(1) = 0$	Since $f'(1) = 0$, then f(x) has a horizontal tangent at $x = 1$.

$5(\sin(\ln(x)))^2$

Derivative = RATE OF CHANGE!

We already understand the derivative to be the **SLOPE OF THE TANGENT LINE**. Slope is a **rate**. Therefore, the derivative of a function actually represents the **RATE AT WHICH A FUNCTION IS CHANGING.**

USE CALCULATOR!

7.	The number of people entering a concert can be modeled by the function $f(t) = 560e^{\sin t}$, where t represents the number of hours after the gates are open.
a.	Find the values of $f\left(\frac{1}{2}\right)$ and $f'\left(\frac{1}{2}\right)$. Using correct units, explain what each value represents in the context of this problem. $f(\frac{1}{2}) = 560e^{\sin(1/2)} = 904.48$; After half an hour 904 people have entered the concert. $f'(\frac{1}{2}) = 793.76$; After half an hour, people are entering the concert at a rate of 793 people per hour.
b.	How many people have entered the concert 2 hours after the gates are opened? Is the number of people entering increasing or decreasing at this time? Justify your answer. $f(2) = 1390.24$; Two hours after the gates opened, 1390 people have entered the Concert. $f'(2) = -578.55$; Since $f'(2) < 0$, then the number of people entering 2 hrs after the gates opening is decreasing at a rate of 578 people per hour.

8.	After being poured into a cup, coffee cools so that its temperature, $T(t)$, is represented by the function $T(t) = 70 + 110e^{-t/2}$, where t is measured in minutes and $T(t)$ is measured in degrees Fahrenheit.
a.	What is the temperature of the coffee 5 minutes after it has been poured into the cup? $T(5) = 79.03$ °F; 5 minutes after the coffee was poured, the temperature is 79.03 °F.
b.	Is the temperature decreasing faster 1 minute after it is poured or 3 minutes after it is poured? Give a reason for your answer. $T'(1) = -33.36$ °F per minute $T'(3) = -12.27$ °F per minute The temperature is decreasing faster 1 minute after the coffee is poured b/c $\lvert T'(1)\rvert > \lvert T'(3)\rvert$.