AP Calculus AB Name: Answer Key*
Unit 3 – Day 2 – Assignment

For exercises 1 and 2, show the algebraic analysis that leads to the derivative of the function. Find the derivative by the specified method.

1. $f(x)=\dfrac{2x^3-3x^2+3}{x^2}$ Rewrite $f(x)$ in a polynomial-form first. Then apply the power rule to find $f'(x)$.	$f(x)=2x-3+3x^{-2}$ $f'(x)=2-6x^{-3}$ $\dfrac{(x^3)}{(x^3)}2-\dfrac{6}{x^3} = \boxed{\dfrac{2x^3-6}{x^3}}$
2. $f(x)=\dfrac{2x^3-3x^2+3}{x^2}$ Apply the quotient rule to find $f'(x)$.	$f'(x)=\dfrac{(x^2)(6x^2-6x)-(2x^3-3x^2+3)(2x)}{(x^2)^2}$ $=\dfrac{6x^4-\cancel{6x^3}-4x^4+\cancel{6x^3}-6x}{x^4}$ $=\dfrac{2x^4-6x}{x^4}=\dfrac{\cancel{x}(2x^3-6)}{\cancel{x}(x^3)}=\boxed{\dfrac{2x^3-6}{x^3}}$

3. Find the equation of the line tangent to the graph of $g(x)=\dfrac{2x^2-3x}{3x+1}$ when $x=-1$.

$$\text{P.O.T} \rightarrow g(-1)=\frac{2(-1)^2-3(-1)}{3(-1)+1}=\frac{2+3}{-3+1}=\frac{5}{-2}\quad (-1,-5/2)$$

$$\text{S.O.T} \rightarrow g'(x)=\frac{(3x+1)(4x-3)-(2x^2-3x)(3)}{(3x+1)^2}$$

$$g'(-1)=\frac{(3(-1)+1)(4(-1)-3)-(2(-1)^2-3(-1))(3)}{[3(-1)+1]^2}$$

$$=\frac{(-2)(-7)-(5)(3)}{4}=\frac{14-15}{4}=\frac{-1}{4}$$

$$\boxed{y+\frac{5}{2}=-\frac{1}{4}(x+1)}$$

Find the derivative of each of the following functions.

4. $h(x) = \dfrac{x}{x^2+1}$

$$h'(x) = \frac{(x^2+1)(1)-(x)(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \boxed{\frac{-x^2+1}{(x^2+1)^2}}$$

5. $h(x) = \dfrac{x}{\sqrt{x}+1} = \dfrac{x}{x^{1/2}+1}$

$$h'(x) = \frac{(x^{1/2}+1)(1)-(x)(\frac{1}{2}x^{-1/2})}{(x^{1/2}+1)^2}$$

$$= \frac{x^{1/2}+1-\frac{1}{2}x^{1/2}}{(x^{1/2}+1)^2}$$

$$= \frac{\frac{1}{2}x^{1/2}+1}{(x^{1/2}+1)^2} = \frac{\frac{1\sqrt{x}}{2}+1}{(\sqrt{x}+1)^2}$$

$$\frac{\sqrt{x}}{2}+\frac{2}{2} = \frac{\sqrt{x}+2}{2} \cdot \frac{1}{(x+1)^2}$$

$$\boxed{\frac{\sqrt{x}+2}{2(x+1)^2}}$$

6. $g(\theta) = \dfrac{\cos\theta}{\theta^3}$

$$g'(\theta) = \frac{(\theta^3)(-\sin\theta)-(\cos\theta)(3\theta^2)}{(\theta^3)^2}$$

$$= \frac{-\theta^3\sin\theta - 3\theta^2\cos\theta}{\theta^6}$$

$$= \frac{-\cancel{\theta^2}(\theta\sin\theta - 3\cos\theta)}{\cancel{\theta^2}\,\theta^4}$$

$$= \boxed{\frac{-(\theta\sin\theta - 3\cos\theta)}{\theta^4}}$$

7. $f(\theta) = \dfrac{3(1-\sin\theta)}{2\cos\theta} = \dfrac{3-3\sin\theta}{2\cos\theta}$

$$f'(\theta) = \frac{(2\cos\theta)(-3\cos\theta)-(3-3\sin\theta)(-2\sin\theta)}{(2\cos\theta)^2}$$

$$f'(\theta) = \frac{-6\cos^2\theta+6\sin\theta-6\sin^2\theta}{(4\cos^2\theta)}$$

$$= \frac{-6(\cos^2\theta - \sin\theta + \sin^2\theta)}{4(\cos^2\theta)}$$

$\cos^2\theta = 1-\sin^2\theta$

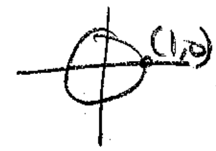
$$= \frac{-6(1-\sin\theta)}{4(1-\sin^2\theta)}$$

$$= \frac{-3(1-\cancel{\sin\theta})}{2(1+\sin\theta)(1-\cancel{\sin\theta})}$$

$$= \boxed{\frac{-3}{2(1+\sin\theta)}}$$

Use the table below to complete exercises 8 – 10.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–2	1	–1	2	4
–1	3	–2	1	1
0	–1	2	–2	–3



8. If $H(x)=\dfrac{2f(x)}{g(x)}$, what is the equation of the tangent line when $x=-1$?

• P.O.T → $H(-1)=\dfrac{2f(-1)}{g(-1)}=\dfrac{2(3)}{(1)}=6$

(-1, 6)

• S.O.T →

$$H'(x)=\frac{g(x)2f'(x)-2f(x)g'(x)}{[g(x)]^2}$$

$$H'(-1)=\frac{g(-1)2f'(-1)-2f(-1)g'(-1)}{[g(-1)]^2}$$

$$=\frac{(1)(2)(-2)-2(3)(1)}{(1)^2}$$

$$=\frac{-4-6}{1}=-10$$

$\boxed{y-6=-10(x+1)}$

9. If $J(x)=\dfrac{3x+\cos x}{f(x)}$, what is the value of $J'(0)$?

$$J'(x)=\frac{f(x)(3-\sin x)-(3x+\cos x)f'(x)}{[f(x)]^2}$$

$$J'(0)=\frac{f(0)(3-\sin 0)-(3(0)+\cos 0)f'(0)}{[f(0)]^2}$$

$$=\frac{(-1)(3)-(1)(2)}{(-1)^2}$$

$$=\frac{-3-2}{1}=\boxed{-5}$$

10. If $K(x)=\dfrac{4x+f(x)}{3-g(x)}$, what is the slope of the normal line when $x=-2$?

$$K'(x)=\frac{(3-g(x))(4+f'(x))-(4x+f(x))(-g'(x))}{[(3-g(x))]^2}$$

$$K'(-2)=\frac{(3-g(-2))(4+f'(-2))-(4(-2)+f(-2))(-g'(-2))}{[3-g(-2)]^2}$$

$$=\frac{(3-4)(4+-1)-(-8+1)(-4)}{(3-2)^2}$$

$$=\frac{(-1)(3)-(-7)(-4)}{(1)}=\frac{-3-28}{1}=-25$$

S.O.T

$\boxed{\text{Slope of normal} = 1/25}$