AP Calculus AB
Unit 3 – Day 1 – Assignment

Name: *Answer Key*

In the table below, a function is given. Show the algebraic analysis that leads to the derivative of the function. Find the derivative by the specified method.

1. $f(x)=(x^2+2x)(x-3)$ Rewrite $f(x)$ as a polynomial first. Then apply the power rule to find $f'(x)$.	$f(x)=x^3-3x^2+2x^2-6x$ $f(x)=x^3-x^2-6x$ $\boxed{f'(x)=3x^2-2x-6}$
2. $f(x)=(x^2+2x)(x-3)$ Apply the product rule to find $f'(x)$.	$f'(x)=(2x+2)(x-3)+(x^2+2x)(1)$ $=2x^2-6x+2x-6+x^2+2x$ $\boxed{f'(x)=3x^2-2x-6}$

For exercises 3 – 5, find the derivative of each function.

3. $f(x)=(x^2+2)(x^2-2x)$

$f'(x)=(2x)(x^2-2x)+(x^2+2)(2x-2)$

$=2x^3-4x^2+2x^3-2x^2+4x-4$

$=\boxed{4x^3-6x^2+4x-4}$

4. $f(x)=(x^3-3x)(2x^2+3x+5)$

$f'(x)=(3x^2-3)(2x^2+3x+5)+(x^3-3x)(4x+3)$

$6x^4+9x^3+15x^2-6x^2-9x-15+4x^4+3x^3-12x^2-9x$

$\boxed{10x^4+12x^3-3x^2-18x-15}$

5. $f(x)=\sqrt[3]{x}(x^2+4)$

$f(x)=x^{1/3}(x^2+4)$

$f'(x)=(\frac{1}{3}x^{-2/3})(x^2+4)+(x^{1/3})(2x)$

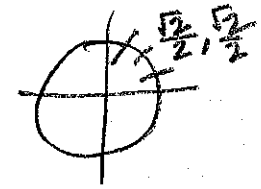
$\frac{1}{3}x^{\frac{4}{3}}+\frac{4}{3}x^{-2/3}+2x^{\frac{4}{3}}$

$\frac{7}{3}x^{\frac{4}{3}}+\frac{4}{3}x^{-2/3}$

$\frac{7x^{4/3}}{3}+\frac{4}{3x^{2/3}}$

$\frac{(\sqrt[3]{x^2})\,7\sqrt[3]{x^4}}{(\sqrt[3]{x^2})\,3}+\frac{4}{3\sqrt[3]{x^2}}$

$\boxed{\frac{7x^2+4}{3\sqrt[3]{x^2}}}$



Find the slope of the normal line drawn to the graph of each function at the indicated value of x.

6. $g(x) = \sqrt{x}\sin x$ when $x = \pi$

$x^{1/2}$

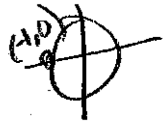
S.O.T $\rightarrow$ $g'(x) = (\frac{1}{2}x^{-1/2})(\sin x) + (x^{1/2})(\cos x)$

$g'(\pi) = \frac{1}{2\sqrt{\pi}}(\sin(\pi)) + \sqrt{\pi}(\cos\pi)$

$\cancel{\frac{1}{2\sqrt{\pi}}(0)} + \sqrt{\pi}(-1)$

S.O.T $= -\sqrt{\pi}$

$\boxed{\text{S.O.N} = \frac{1}{\sqrt{\pi}}}$



7. $h(x) = \sin x(\sin x + \cos x)$ when $x = \frac{\pi}{4}$

$h'(x) = (\cos x)(\sin x + \cos x) + (\sin x)(\cos x - \sin x)$

$h'(\frac{\pi}{4}) = (\cos\frac{\pi}{4})(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}) + (\sin\frac{\pi}{4})(\cos\frac{\pi}{4} - \sin\frac{\pi}{4})$

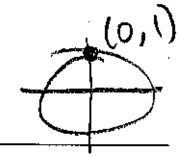
$= (\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) + \cancel{(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2})}$

$= (\frac{\sqrt{2}}{2})(\frac{2\sqrt{2}}{2})$

$= \frac{2(2)}{4} = \frac{4}{4} = 1$

$\boxed{\text{S.O.N} = -1}$

For each of the functions below, find the equation of the tangent line drawn to the graph of $g(x)$ at the indicated value of x.



8. $g(x) = \sqrt{x}(2x^2 - 4)$ when $x = 4$

P.O.T $\rightarrow$ $g(4) = \sqrt{4}(2(4)^2 - 4))$

$= (2)(28)$

$= 56$ (4,56)

S.O.T $\rightarrow$ $g(x) = x^{1/2}(2x^2 - 4)$

$g'(x) = \frac{1}{2}x^{-1/2}(2x^2 - 4) + x^{\frac{1}{2}}(4x)$

$= \frac{1}{2\sqrt{x}}(2x^2 - 4) + \sqrt{x}(4x)$

$g'(4) = \frac{1}{2\sqrt{4}}(2(4)^2 - 4) + \sqrt{4}(4(4))$

$\frac{1}{4}(28) + 32$

$= 39$

$\boxed{y - 56 = 39(x - 4)}$

9. $g(x) = x^2\cos x$ when $x = \frac{\pi}{2}$

P.O.T

$g(\frac{\pi}{2}) \rightarrow (\frac{\pi}{2})^2\cos(\frac{\pi}{2}) = \frac{\pi^2}{4}(0) = 0$

$(\frac{\pi}{2}, 0)$

S.O.T $g'(x) = (2x)(\cos x) + (x^2)(-\sin x)$

$g'(\frac{\pi}{2}) = \cancel{2(\frac{\pi}{2})}\cos\frac{\pi}{2} + (\frac{\pi}{2})^2(-\sin\frac{\pi}{2})$

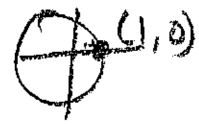
$\cancel{\pi(0)} + \frac{\pi^2}{4}(-1)(1)$

$-\frac{\pi^2}{4}$

$\boxed{y = -\frac{\pi^2}{4}(x - \frac{\pi}{2})}$

Use the table below to complete exercises 10 – 12.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–2	1	–1	2	4
–1	3	–2	1	1
0	–1	2	–2	–3



10. If $H(x) = 2f(x)\cdot g(x)$, what is the equation of the tangent line when $x = -1$?

•P.O.T → $H(-1) = 2f(-1)\cdot g(-1)$

$= 2(3)\cdot(1) = 6$

$(-1, 6)$

•S.O.T → $H'(x) = (2f'(x))(g(x)) + 2f(x)g'(x)$

$H'(-1) = 2f'(-1)g(-1) + 2f(-1)g'(-1)$

$= 2(-2)(1) + 2(3)(1)$

$= -4 + 6 = 2$

$\boxed{y - 6 = 2(x+1)}$

11. If $J(x) = g(x)\cdot \sin x$, what is the value of $J'(0)$?

$J'(x) = (g'(x))(\sin x) + (g(x))(\cos x)$

$J'(0) = g'(0)\sin(0) + g(0)\cos(0)$

$(-3)(0) + (-2)(1)$

$0 - 2 = \boxed{-2}$

12. If $K(x) = (4x - f(x))(2g(x) - 2)$, what is the slope of the normal line when $x = -2$?

$K'(x) = (4 - f'(x))(2g(x) - 2) + (4x - f(x))(2g'(x))$

$K'(-2) = (4 - f'(-2))(2g(-2) - 2) + (4(-2) - f(-2))(2g'(-2))$

$= (4 - -1)(2(2) - 2) + (-8 - 1)(2(4))$

$(5)(2) + (-9)(8)$

$10 - 72 = -62$ S.O.T

$\boxed{\text{S.O.N} \rightarrow \frac{1}{62}}$