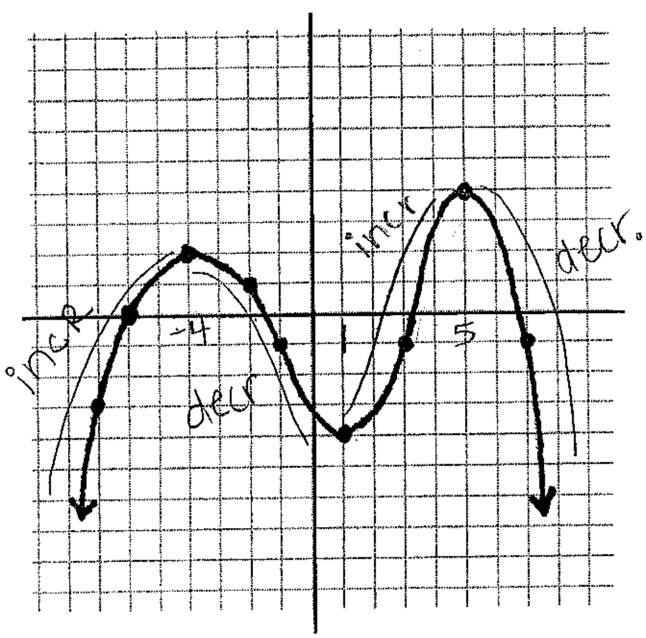
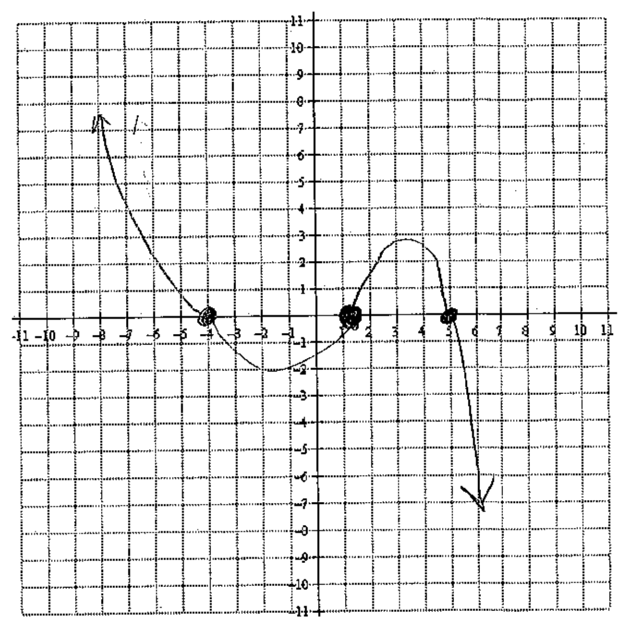
AP Calculus
Unit 2 – Conceptualizing the Derivative

Day 4 Notes: Connections Between F(x) and F'(X) for Polynomial & Trigonometric Functions

If $F'(x)$…	then $F(x)$…
…is = 0,	horizontal tangent line (possibly rel. max or rel. min)
…is > 0,	increasing
…is < 0,	decreasing
…changes from positive to negative,	relative maximum
…changes from negative to positive,	relative minimum



Graph of f(x)



Possible Graph of $f'(x)$

Example 1: For each of the given functions, determine the interval(s) on which $f(x)$ is increasing and/or decreasing. Find all coordinates of the relative extrema. Unless otherwise noted, perform the analysis on all values on $(-\infty, \infty)$. Provide justification for your answers.

$f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2$

$15x^4 - 15x^2 = 0$

$15x^2(x^2-1) = 0$

$15x^2(x+1)(x-1) = 0$

$x = 0,\ x = -1,\ x = 1$

$-2 \quad -.5 \quad .5 \quad 2$

$f'(x)$: $+ \quad (-1) \quad - \quad (0) \quad - \quad (1) \quad +$

$(+)(-)(-) \quad (+)(+)(-) \quad (+)(+)(-) \quad (+)(+)(+)$

$f'(x) > 0$ ① increasing → $(-\infty, -1) \cup (1, \infty)$

$f'(x) < 0$ ② decreasing → $(-1, 0) \cup (0, 1)$

$f'(x) = 0$ & $\oplus \to \ominus$

$x = -1$ is rel. max

$f(-1) = 3(-1)^5 - 5(-1)^3 = -3 + 5 = 2$

③ $(-1, 2)$ is rel. max

$f'(x) = 0$, $\ominus \to \oplus$

$x = 1$ is rel. min

$f(1) = 3(1)^5 - 5(1)^3 = -2$

④ $(1, -2)$ is rel. min

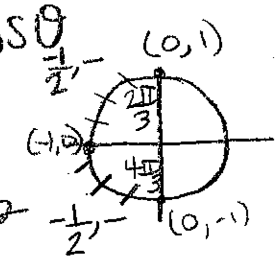
$f(\theta) = \theta + 2\sin\theta$ on $(0, 2\pi)$

$f'(\theta) = 1 + 2\cos\theta$

$1 + 2\cos\theta = 0$

$2\cos\theta = -1$

$\cos\theta = -1/2$



$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\pi/2 \quad \pi \quad 3\pi/2$

$0 \quad + \quad \frac{2\pi}{3} \quad - \quad \frac{4\pi}{3} \quad + \quad 2\pi$

$1+2\cos(\frac{\pi}{2})$, $1+0$ $\quad$ $1+2\cos(\pi)$, $1+-2$ $\quad$ $1+2\cos(\frac{3\pi}{2})$, $1+0$

① increasing → $(0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi)$

② decreasing → $(2\pi/3, 4\pi/3)$

rel. max at $x = \frac{2\pi}{3}$

$f(\frac{2\pi}{3}) = \underbrace{\frac{2\pi}{3} + 2(\sin(\frac{2\pi}{3}))}_{\text{use calc.}} \approx 3.826$

③ rel. max = $(2\pi/3, 3.826)$

rel. min at $x = \frac{4\pi}{3}$

$f(4\pi/3) = \frac{4\pi}{3} + 2\sin(\frac{4\pi}{3}) \approx 2.457$

④ rel. min at $(4\pi/3, 2.457)$

under approx. ← concave up = slope increases 

over approx. ← concave down = slope decreases

Example 2: The table of values below represents values on the graph of the derivative, $h'(x)$, of a polynomial function $h(x)$. The zeros indicated in the table are the only zeros of the graph of $h'(x)$. Additionally, the graph of $h(x)$ is **concave up** at $x = 3$. Use the table to answer questions ~~10 – 15~~. (a – e)

⊕ ⊖ ⊖

x	-8	-5	-2	0	3	5	7	10	12
$h'(x)$	11	5	0	-1	-3	-1	0	-3	-9

a) On what interval(s) is the function $h(x)$ increasing and decreasing? Give reasons for your answers.

increasing → $(-\infty, -2)$

decreasing → $(-2, 7) \cup (7, \infty)$

b) At what x – value(s) does the graph of $h(x)$ have a relative maximum? Justify your answer.

$x = -2$

c) At what x – value(s) does the graph of $h(x)$ have a relative minimum? Justify your answer.

no relative minimum → no change from ⊖ to ⊕

d) If $h(3) = 2$, what is the equation of the tangent line to the graph of $h(x)$ at $x = 3$? What is the equation of the normal line to the graph of $h(x)$ at $x = 3$?

point $(3, 2)$

$h'(3) = -3$ slope

$\boxed{y - 2 = -3(x - 3)}$ Tangent

normal slope = $1/3$

$\boxed{y - 2 = \frac{1}{3}(x - 3)}$ Normal

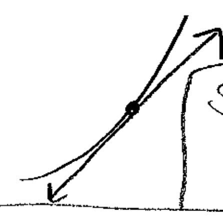
* e) Find the tangent line approximation of $h(3.1)$. Is this approximation greater or less than the actual value of $h(3.1)$? Give a reason for your answer.

$h(3.1) \approx y - 2 = -3(3.1 - 3)$

$y - 2 = -3(.1)$

$y - 2 = -.3$

$\boxed{y = 1.7}$



since concave up at $x = 3$, then

$\boxed{\text{under approximation}}$