AP Calculus AB
Unit 2 – Day 3 – Assignment

Name: Answer Key*

For exercises 1 – 12, find the derivative of each function. Leave your answers with no negative or rational exponents and as single rational functions, when applicable.

1. $f(x) = 5 - 2x^2 - 3x^3$ $\boxed{f'(x) = -4x - 9x^2}$	2. $h(x) = \dfrac{2x^3 + 3x^2 - 2x}{x}$ $h(x) = 2x^2 + 3x - 2$ $\boxed{h'(x) = 4x + 3}$
3. $h(x) = \dfrac{3}{x^7}$ $h(x) = 3x^{-7}$ $h'(x) = -21x^{-8}$ $\boxed{h'(x) = \dfrac{-21}{x^8}}$	4. $g(x) = \dfrac{2x^5}{x^8}$ $g(x) = 2x^{-3}$ $g'(x) = -6x^{-4}$ $\boxed{g'(x) = \dfrac{-6}{x^4}}$
5. $f(\theta) = -3\theta^2 - \cos\theta$ $f'(\theta) = -6\theta - (-\sin\theta)$ $\boxed{f'(\theta) = -6\theta + \sin\theta}$	6. $h(x) = \sqrt[3]{x^2}$ $h(x) = x^{2/3}$ $h'(x) = \frac{2}{3}x^{-1/3}$ $\frac{2}{3x^{1/3}}$ $\boxed{h'(x) = \dfrac{2}{3\sqrt[3]{x}}}$
7. $g(\theta) = \sqrt{\theta} + 2\sin\theta$ $g(\theta) = \theta^{1/2} + 2\sin\theta$ $g'(\theta) = \frac{1}{2}\theta^{-1/2} + 2(\cos\theta)$ $\dfrac{1}{2\sqrt{\theta}} + \dfrac{2\cos\theta}{1}\dfrac{(2\sqrt{\theta})}{(2\sqrt{\theta})}$ $\boxed{g'(\theta) = \dfrac{1 + 4\sqrt{\theta}\cos\theta}{2\sqrt{\theta}}}$	8. $p(x) = -2x^{\frac{3}{2}} + \sqrt{x}$ $p(x) = -2x^{3/2} + x^{1/2}$ $p'(x) = -3x^{1/2} + \frac{1}{2}x^{-1/2}$ $\dfrac{-3\sqrt{x}}{1}\dfrac{(2\sqrt{x})}{2(\sqrt{x})} + \dfrac{1}{2\sqrt{x}}$ $\boxed{p'(x) = \dfrac{-6x + 1}{2\sqrt{x}}}$

9. $g(x) = (x+3)(2x-1)^2$

$(X+3)(2X-1)(2X-1)$

$2X^2-X+6X-3$

$(2X^2+5X-3)(2X-1)$

$4X^3-2X^2+10X^2-5X-6X+3$

$g(x)=4X^3+8X^2-11X+3$

$\boxed{g'(x)=12X^2+16X-11}$

10. $h(x) = \dfrac{x^2+2x-2}{x^3}$

$h(x) = X^{-1}+2X^{-2}-2X^{-3}$

$h'(x) = -1X^{-2}-4X^{-3}+6X^{-4}$

$\dfrac{-1(x^2)}{X^2(x^2)} - \dfrac{4(x)}{X^3(x)} + \dfrac{6}{X^4}$

$\boxed{h'(x) = \dfrac{-X^2-4X+6}{X^4}}$

11. $f(x) = \dfrac{3x}{\sqrt[3]{x}}$

$\dfrac{3X}{X^{1/3}}$

$f(x) = 3X^{2/3}$

$f'(x) = 2X^{-1/3}$

$= \dfrac{2}{X^{1/3}}$

$\boxed{f'(x) = \dfrac{2}{\sqrt[3]{X}}}$

12. $h(x) = 6\sqrt{x} - 3\cos x$

$6X^{1/2} - 3\cos X$

$h'(x) = 3X^{-1/2} - 3(-\sin x)$

$\dfrac{3}{\sqrt{X}} + \dfrac{3\sin x\,(\sqrt{x})}{1\,(\sqrt{x})}$

$\boxed{h'(x) = \dfrac{3+3\sqrt{x}\sin x}{\sqrt{x}}}$

13. For what value(s) of x will the slope of the tangent line to the graph of $h(x) = 4\sqrt{x}$ be 2? slope
Find the equation of the line tangent to $h(x)$ at this/these x – values. Show your work.

$h'(x) = 2?$

$h(x) = 4x^{1/2}$

$h'(x) = 2x^{-1/2}$

$h'(x) = \dfrac{2}{\sqrt{x}}$

$\dfrac{2}{\sqrt{x}} = 2$

$2 = 2\sqrt{x}$

$1 = \sqrt{x}$

$\boxed{X=1}$

P.O.T

$h(1) = 4\sqrt{1}$

$(1,4)$

$\boxed{Y-4=2(X-1)}$

14. Find the equation of the line tangent to the graph of $g(x) = \dfrac{2}{\sqrt[4]{x^3}}$ when $x = 1$.

$g(x) = \dfrac{2}{X^{3/4}}$

$g(x) = 2X^{-3/4}$

$g'(x) = -\frac{3}{2}X^{-7/4}$

$g'(1) = -\frac{3}{2}(1)^{-7/4} = -3/2$ slope

P.O.T $g(1) = \dfrac{2}{\sqrt[4]{1^3}} = 2$ $(1,2)$

$\boxed{Y-2=-\frac{3}{2}(X-1)}$

15. The line defined by the equation $\frac{1}{2}x + 3 = -2(y-3)$ is the line tangent to the graph of a function $f(x)$ when $x = a$. What is the value of $f'(a)$? Show your work and explain your reasoning.

slope of tangent

$$\frac{1}{2}x+3 = -2y+6$$

$$\frac{\frac{1}{2}x-3 = -2y}{-2}$$

$$-\frac{1}{4}x+\frac{3}{2} = y$$

$$\boxed{f'(a) = -1/4}$$

16. The line defined by the equation $y - 3 = -\frac{2}{3}(x+3)$ is the line tangent to the graph of a function $f(x)$ at the point $(-3, 3)$. What is the equation of the normal line when $x = -3$. Explain your reasoning.

$$y-3 = -\frac{2}{3}x-2$$

$$y = -\frac{2}{3}x+1$$

Slope of tangent

Slope of normal $= \frac{3}{2}$

$$\boxed{y-3 = \frac{3}{2}(x+3)}$$

17. Determine the value(s) of x at which the function $f(x) = x^4 - 8x^2 + 2$ has a horizontal tangent.

$f'(x) = 0$

$$f'(x) = 4x^3 - 16x$$

$$4x^3 - 16x = 0$$

$$4x(x^2-4) = 0$$

$$4x(x+2)(x-2) = 0$$

$$\boxed{x=0} \quad \boxed{x=-2} \quad \boxed{x=2}$$

18. Determine the value(s) of θ at which the function $f(\theta) = \sqrt{3}\theta + 2\cos\theta$ has a horizontal tangent on the interval $[0, 2\pi)$.

$f'(\theta) = 0$

$$f'(\theta) = \sqrt{3} + 2(-\sin\theta)$$

$$f'(\theta) = \sqrt{3} - 2\sin\theta$$

$$\sqrt{3} - 2\sin\theta = 0$$

$$-2\sin\theta = -\sqrt{3}$$

$$\sin\theta = +\frac{\sqrt{3}}{2}$$

$$\boxed{\theta = \frac{\pi}{3}, \ \theta = \frac{2\pi}{3}}$$

19. For what value(s) of k is the line $y = 4x - 9$ tangent to the graph of $f(x) = x^2 - kx$?

slope = 4

$$f'(x) = 2x - k$$

$$2x - k = 4$$

$$2x - 4 = k$$

$$x^2 - (2x-4)(x) = 4x - 9$$

$$x^2 - 2x^2 + 4x = 4x - 9$$

$$x^2 - 2x^2 + 9 = 0$$

$$-x^2 + 9 = 0$$

$$x^2 - 9 = 0 \quad x = \pm 3$$

$$k = 2(-3) - 4 = \boxed{-10}$$

$$k = 2(3) - 4 = \boxed{2}$$