AP Calculus
Unit 2 – Conceptualizing the Derivative

Day 2 Notes: Understanding the Derivative from a Graphical & Numerical Approach

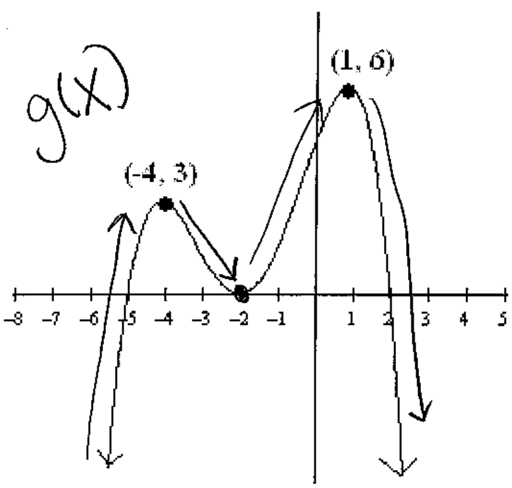
Graphical behaviors of *f(x)* based on the value of the derivative, *f'(x)*:

1) If $f'(a) > 0$ (positive), then the graph of *f(x)* is INCREASING at x = a.

2) If $f'(a) < 0$ (negative), then the graph of *f(x)* is DECREASING at x = a.

3) *f(x)* has a relative maximum if $f'(a) = 0$ and *f'(x)* changes from positive (incr) to negative (decr).
   incr decr

4) *f(x)* has a relative minimum, if $f'(a) = 0$ and *f'(x)* changes from negative (decr.) to positive (incr.).
   decr incR.



Example 1: The graph of a function, *g*(*x*), is pictured to the left. Identify the following characteristics about the graph of the derivative, *g'*(*x*). Give a reason for your answers.

The interval(s) where $g'(x) < 0$ g'(x) is negative	g'(x) is negative when g(x) is decreasing. $(-4,-2) \cup (1,\infty)$
The interval(s) where $g'(x) > 0$ g'(x) is positive	g'(x) is positive when g(x) is increasing $(-\infty,-4) \cup (-2,1)$
The value(s) of x where $g'(x) = 0$	g'(x) = 0 when we have relative maximum or rel. minimum. $x=-4,\ x=-2,\ x=1$

*use the x-values for your intervals

Estimating the Derivative:
Numerically, the value of the derivative at a point can be **ESTIMATED** by finding the SLOPE of the SECANT LINE passing through two points on the graph on either side of the point for which the derivative is being estimated.

Example 2: Use the table below to fill out the chart.

x	−3	0	1	4	6	10
$f(x)$	2	1	−3	0	−7	2

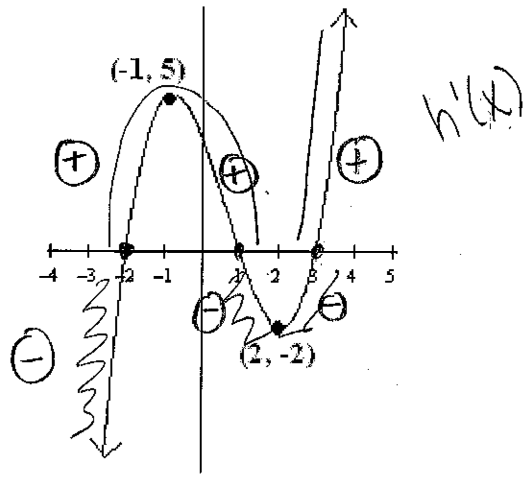
x – Value	Estimation of Derivative	Is the function Increasing, Decreasing or at a Relative Maximum or Relative Minimum	Equation of the tangent line at this value of x.
0	(−3, 2) & (1, −3) $f'(0) \approx \frac{-3-2}{1--3} \approx \frac{-5}{4}$	decreasing	P.O.T → (0, 1) S.O.T ≈ −5/4 $\boxed{y-1=-\frac{5}{4}(x)}$
1	(0, 1) & (4, 0) $f'(1) \approx \frac{0-1}{4-0} \approx \frac{-1}{4}$	decreasing	P.O.T → (1, −3) S.O.T ≈ −1/4 $\boxed{y+3=-\frac{1}{4}(x-1)}$
4	(1, −3) & (6, −7) $f'(4) \approx \frac{-7--3}{6-1} \approx -\frac{4}{5}$	decreasing	P.O.T → (4, 0) S.O.T ≈ −4/5 $\boxed{y=-\frac{4}{5}(x-4)}$
6	(4, 0) & (10, 2) $f'(6) \approx \frac{2-0}{10-4} \approx \frac{2}{6} \approx \frac{1}{3}$	Increasing	P.O.T → (6, −7) S.O.T ≈ 1/3 $\boxed{y+7=\frac{1}{3}(x-6)}$

Definition of the Normal Line:
The normal line is the line that is **perpendicular to the tangent line** at the point of tangency.

↓

opposite reciprocal slope

Example 3: The graph of the derivative, $h'(x)$, of a function $h(x)$ is pictured below. Identify the following characteristics about the graph of $h(x)$ and give a reason for your responses.



The interval(s) where $h(x)$ is increasing	If $h(x)$ is increasing, then $h'(x) > 0$. If $h'(x) > 0$, then its graph is ABOVE the x-axis. $\boxed{(-2, 1) \cup (3, \infty)}$
The interval(s) where $h(x)$ is decreasing	If $h(x)$ is decreasing, then $h'(x) < 0$. If $h'(x) < 0$, then the graph is BELOW the x-axis $\boxed{(-\infty, -2) \cup (1, 3)}$
The value(s) of x where $h(x)$ has a relative maximum.	$h(x)$ has rel. max when $h'(x) = 0$ and $h'(x)$ changes from positive to negative. $\boxed{x = 1}$
The value(s) of x where $h(x)$ has a relative minimum.	$h(x)$ has rel. min when $h'(x) = 0$ and $h'(x)$ changes from negative to positive $\boxed{x = -2 \text{ \& } x = 3}$
If $h(-1) = ½$, what is the equation of the tangent line drawn to the graph of $h(x)$ at $x = -1$?	P.O.T $\rightarrow (-1, 1/2)$ S.O.T $\rightarrow h'(-1) = 5$ (from graph) $\boxed{y - \frac{1}{2} = 5(x+1)}$
If $h(2) = -3$, what is the equation of the normal line drawn to the graph of $h(x)$ at $x = 2$?	P.O.T $\rightarrow (2, -3)$ S.O.T $\rightarrow h'(2) = -2$ (from graph) slope of normal $= 1/2$ $\boxed{y + 3 = \frac{1}{2}(x-2)}$